# Advanced Macroeconomics
# –
# DSGE modeling – Second generation of models (New Keynesian models)

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# RBC VS New Keynesian models

next

| RBC | New Keynesian |
|---|---|
| Microfoundation | Accepted |
| General equilibrium | Accepted |
| Flexible prices | Rejected! → Nominal rigidities |
| All market are frictionless | Rejected! → Monopolistic competition |
| | Equilibrium production < social optimum |
| Neutrality of monetary policy | Rejected! |
| Only real variables matter | Short-term nominal effects |
| Efficiency of the business cycle | Monetary policy corrects distortions |
| Fiscal policy is distortionary | The multiplier **usually** $< 1$ |
| Long-run description of dynamics | Shorter-run description of dynamics |

# Nominal rigidities

**Why are prices sticky?**

- There is the cost of physically changing posted prices ("menu" costs).
- A firm that changes its prices may face a cost which results from the negative reaction of its costumers (reputation loss).
- State-dependent pricing rules: firms adjust more often and more drastically in a boom than in a downturn (because competition for demand is stronger).
- Information frictions (Mankiw & Reis 2005): heterogeneity in the degree of exposure to and diffusion of the information (time to react to a news).

# Nominal rigidities

**Why are nominal wages sticky?**

- Nominal wages are typically set by contract, in advance (downward nominal wage rigidities).
- Negative effect on workers of a decrease in nominal wages (political pressures, trade unions).
- Information frictions (just like for the prices).
- **Downward** nominal rigidities lead to **upward** nominal rigidities.

# Nominal rigidities

**Different modeling approaches:**

1. **Quadratic adjustment costs** (Rotemberg, 1982): account for the negative effects of price changes on the customer-firm relationship (increases as the size of the price change increases) through the utility function of a representative firm-household:

$$U_{t,j}\left(c_{t,j}, h_{t,j}, \frac{P_t}{P_{t-1}} - 1\right) \equiv \frac{c_{t,j}^{1-\sigma}}{1-\sigma} - \frac{h_{t,j}^{1+\epsilon}}{1+\epsilon} - \frac{\gamma}{2}\left(\frac{P_{t,j}}{P_{t-1,j}} - 1\right)^2$$

   $\gamma$ represents the cost of inflation.

2. **Time dependent pricing models**:
   - Calvo fixed probability of adjusting each period (see below);
   - Taylor's model of fixed-length contracts

3. State dependent price models (stronger and "cheaper" adjustments in booms than in downturns): the costs of adjustments depends on the state of the economy ($\gamma = \gamma(y_t)$).

## Implications

1. **Short-run**, nominal rigidities make the firms **forward-looking**, i.e. they make **intertemporal decisions**, like the households (in RBC models, the firms' decision rules were only intratemporal).
   $\rightarrow E(\cdot)$ in their behavioral rules from the FOC.

2. Nominal rigidities make the Phillips curve **downward-sloping in the short-run** and **forward-looking**: **New Keynesian Phillips Curve** (NKPC).

3. Nominal rigidities are necessary to obtain **real effects of policies**, especially monetary policy (**short-term** effects).

# Nominal rigidities

## Empirical evidence

- **Mixed empirical evidence** in favor of models of price stickiness: Klenow and Kryvtsov (QJE, 2008)
- **Micro facts on the median duration between price changes:**
  - For the U.S. (Bils and Klenow, JPE 2004): 4.3 months for items.
  - For the EA (Altissimo, Ehrmann and Smets, NBB WP 2006): $4-5$ quarters, but there is a substantial degree of heterogeneity across products and sectors.

# The households

## Assumptions of the NK models wrt the RBC models

- A representative household (or a continuum from 0 to 1) maximizes its **expected** utility function over an infinite horizon (same as in RBC models);
- consumes a composite good $C$, which is an aggregation of differentiate goods indexed by $j \in [0, 1]$ (**monopolistic/imperfect competition**);
- holds the firms and receive an equal share of the **net profits** (as market power, no price taking behavior), capital is constant (short-run view);
- supplies a quantity $L$ of labor (in hours) and receive a **nominal** hourly wage $W$;
- holds bonds (Treasury bonds) $B$ that yield a **nominal** interest rate $i$ and a quantity of money $M$ (understood as cash-on-hand) ($\rightarrow$ nominal variables are introduced).

# The households

**Two step maximization program**:

1. Minimization of the costs of purchasing a quantity $C_t$ of the composite good:

$$\min_{c_{j,t}} \int_0^1 p_{j,t} c_{j,t} dj \Leftrightarrow \min_{c_{j,t}} P_t C_t$$

$$\text{s/c} \quad \left[ \int_0^1 c_{j,t}^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \equiv C_t$$

with $\theta > 1$ the elasticity of substitution between the differentiate goods (the higher $\theta$, the closer substitutes goods are, and the less market power of the firms).

**Demand function for variety $j$**

Inserting the composite index $C_t$ into $C_t P_t$ , taking the first derivative of

$$\int_0^1 p_{j,t} c_{j,t} dj - P_t C_t$$

wrt. $c_{j,t}$ (it can be shown easily that this is an optimum), and rearranging the terms, we get the demand $c_j$ for the differentiate good $j$, as a function of the aggregate price $P$ and the individual price of the goods $j$, $P_j$:

$$\boxed{c_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\theta} C_t} \tag{1}$$

# The households

2 choose $C_t$, $L_t$ and $M_t$ under budget constraint:

$$\max_{C_t, L_t, M_t} E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( \underbrace{\frac{\sigma}{\sigma - 1} C_t^{\frac{\sigma-1}{\sigma}}}_{\text{utility from consumption}} + \underbrace{\frac{1}{1-b} \left( \frac{M_t}{P_t} \right)^{1-b}}_{\text{utility from real money balances}} - \underbrace{\frac{1}{1+\eta} L_t^{1+\eta}}_{\text{disutility from labor}} \right) \right]$$

$$\text{s/c} \quad C_t P_t + M_t + B_t = W_t L_t + M_{t-1} + (1 + i_{t-1}) B_{t-1} + \Pi_t$$

with (as in RBC models) $\beta$ the discount factor, $\sigma > 0$ the intertemporal elasticity of substitution (the lower $\sigma$, the stronger the preference for consumption smoothing), $b > 0$ and $\eta > 0$ some preference (fixed, **policy-invariant**) parameters.

The Lagrangian reads as follows:

$$\max_{C_t, L_t, N_t, B_t} E_0 \left( \sum_{t=0}^{\infty} \beta^i \left( \frac{\sigma}{\sigma - 1} C_t^{\frac{\sigma-1}{\sigma}} + \frac{1}{1-b} \left( \frac{M_t}{P_t} \right)^{1-b} - \frac{\chi}{1+\eta} L_t^{1+\eta} \right) \right.$$
$$\left. + \lambda_t \left( C_t P_t + M_t + B_t - W_t L_t - M_{t-1} - (1 + i_{t-1}) B_{t-1} \right) \right)$$

The FOC are:

$$\begin{cases} \dfrac{\partial \mathcal{L}_t}{\partial C_t} = 0 \Leftrightarrow C_t^{-1/\sigma} + \lambda_t P_t = 0 & \text{(2a)} \\ \dfrac{\partial \mathcal{L}_t}{\partial B_t} = 0 \Leftrightarrow \lambda_t - \beta E_t(\lambda_{t+1})(1 + i_t) = 0 & \text{(2b)} \\ \dfrac{\partial \mathcal{L}_t}{\partial M_t} = 0 \Leftrightarrow \left( \dfrac{M_t}{P_t} \right)^{-b} \dfrac{1}{P_t} + \lambda_t - \beta E_t(\lambda_{t+1}) = 0 & \text{(2c)} \\ \dfrac{\partial \mathcal{L}_t}{\partial L_t} = 0 \Leftrightarrow -L_t^{\eta} - \lambda W_t = 0 & \text{(2d)} \end{cases}$$

# The households

- **Intertemporal optimality** – Using (2b) to express $\lambda_t$, then iterating forward (2a) to substitute for $\lambda_{t+1}$ in (2b) and equalizing $\lambda_t$ with (2a), we get the Euler equation:

$$\boxed{C_t^{-1/\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}C_{t+1}^{-1/\sigma}\right)} \tag{3}$$

- **Intratemporal optimality on the labor market** – Equalizing $\lambda_t$ in (2a) and (2d) gives:

$$\boxed{\frac{L_t^{\eta}}{C_t^{-1/\sigma}} = \frac{W_t}{P_t}} \tag{4}$$

# The households

- **Intratemporal optimality on the money market** – Inserting (2b) into (2c) to replace $\lambda_{t+1}$ by $\lambda_t$, and equalizing $\lambda_t$ with (2a) gives:

$$\boxed{\frac{\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-1/\sigma}} = \frac{i_t}{1+i_t}} \tag{5}$$

- **Transversality conditions**:

$$\lim_{T\longrightarrow\infty} E_t\left(\lambda_{t+T}\frac{B_{t+T+1}}{P_{t+T}}\right) = 0$$

# The firms

- A continuum of firms, indexed by $j$, each producing a differentiate good $c_j$ (monopolistic competition à la Dixit-Stiglitz 1977, Blanchard & Kiyotaki 1987),
- Firms sell goods to the households and are owned by the households,
- Firms have all the same production function with fixed capital ($K_t \equiv \bar{K} = 1$):

$$c_{j,t} = Z_t L_{j,t}^{1-\alpha} \quad \forall j \tag{6}$$

- and same (labor) costs $W_t L_{j,t}$,
- and a **specific demand function** from (1): $c_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\theta} C_t$.

## The firms

**A two-step maximization program**:

1. Minimization of the production costs of a given quantity $c_{j,t}$:

$$\min_{L_t} \frac{W_t}{P_t} L_t$$

$$\text{s/c} \quad c_{j,t} = Z_t L_t^{1-\alpha}$$

Equalizing the derivative of the real costs (**real marginal cost**) and the derivative of the production function wrt. $L_t$, we define the **real marginal cost** $\varphi$:

$$Z_t \cdot \underbrace{(1-\alpha) L_t^{-\alpha}}_{\text{marginal productivity of labor}} = \frac{W_t}{P_t} \Leftrightarrow \boxed{\varphi_t \equiv \frac{W_t}{P_t Z_t}} \tag{7}$$

# Monopolistic competition under flexible prices

**2a** Maximization of profits in the **benchmark of flexible prices**:

$$\max_{p_{j,t}} \Pi_{j,t} = c_{j,t}\left(p_{j,t} - \varphi_t P_t\right) = \left(\frac{p_{j,t}}{P_t}\right)^{-\theta} C_t \left(p_{j,t} - \frac{W_t}{Z_t}\right)$$

(by using (1) and (7) and expressing the profits in nominal terms).

The FOC wrt $p_{j,t}$ gives the price-setting behavior:

$$\frac{p_{j,t}}{P_t} = \underbrace{\frac{\theta}{\theta - 1}}_{>1 \text{ mark-up on the marginal cost}} \varphi_t$$

# Monopolistic competition under flexible prices

All firms face the same cost function and production function.
At the **symmetric** equilibrium, $p^*_{j,t} = p^*_t$, $\forall j$ and $P^*_t = P_t$, then:

$$\frac{P^*_t}{P_t} = 1 = \frac{\theta}{\theta - 1}\varphi_t \equiv \mu\varphi_t \quad \Leftrightarrow \quad \boxed{\varphi_t = \frac{1}{\mu}} \tag{8}$$

### Under flexible prices

The firms solve an intratemporal problem. The price is set so that there is a mark-up $\mu > 1$ over the nominal marginal cost (perfect competition would give $\mu = 1$ or $\theta \to \infty$).

# Monopolistic competition under sticky prices

## 2b Maximization of profits under sticky prices:

- Time-dependent price rigidities (Calvo 1983): a share $(1-w)$ of firms can adjust their price in $t$.
- A price set in $t$ continues to apply in $t+1$ with probability $w$, ... in $t+j$ with probability $w^j$, etc.
- A firm $j$ sets $p_{j,t}$ to maximize the current (in $t$) and future expected profits that are affected by $p_{j,t}$

Price rigidities make the firms forward-looking.

Let us define the discount factor of the firms between periods $t$ and $t+i$ in real terms (inflation): more on discounting with inflation

$$\Delta_{t,t+n+1} = \prod_{k=0}^{n} \frac{1}{1+i_{t+k}} \frac{(P_{t+k+1})}{P_t} = \beta^{n+1} \frac{C_{t+n+1}}{C_t}^{-1/\sigma}$$

# Monopolistic competition under sticky prices

Then, we have (from the Euler equation (3)):

$$\max_{p_{j,t}} E_t \sum_{i=0}^{\infty} w^i \Delta_{i,t+i} \left[ c_{j,t+i} \left( \frac{p_{j,t}}{P_{t+i}} - \varphi_{t+i} \right) \right]$$

The FOC wrt $p_{j,t}$ gives:

$$\boxed{\frac{p_{j,t}}{P_t} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{i=0}^{\infty} w^i \beta^i C_{t+i}^{1-1/\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^{\theta}}{E_t \sum_{i=0}^{\infty} w^i \beta^i C_{t+i}^{1-1/\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\theta-1}}} \qquad (9)$$

where the aggregate price level is defined by the Dixit-Stiglitz index:

$$P_t = \left[ \int_0^1 p_{j,t}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

# Monopolistic competition under sticky prices

$$
\begin{aligned}
P_t^{1-\theta} &= \int_0^{1-w} p_{j,t}^{*1-\theta} dj + \int_{1-w}^{1} P_{t-1}^{1-\theta} dj \\
&= \boxed{(1-w)P_t^{*1-\theta} + wP_{t-1}^{1-\theta}} \qquad (10)
\end{aligned}
$$

→ the aggregate price level $P_t$ is an average of the optimally adjusted prices in $t$ and the non-adjusted prices (i.e. the average of the last period's prices).

It is convenient to define $Q_t \equiv \frac{p_{j,t}}{P_t} = \frac{P_t^*}{P_t}$, the price set by the firms that can adjust their price in $t$ relative to the aggregate price level. $Q_t$ is given by the FOC (9).

# The non-linear version of the model

$$
\begin{cases}
C_t^{-\sigma} = \beta(1+i_t)E_t\left(\frac{P_t}{P_{t+1}}C_{t+1}^{-\sigma}\right) & \text{Euler equation} \\
\frac{\left(\frac{M_t}{P_t}\right)^{-b}}{C_t^{-1/\sigma}} = \frac{i_t}{1+i_t} & \text{money demand} \\
\frac{N_t^{\eta}}{C_t^{-1/\sigma}} = \frac{W_t}{P_t} & \text{labor supply} \\
\frac{W_t}{P_t} = \frac{Z_t}{\mu} & \text{labor demand} \\
Y_t = Z_t L_t^{1-\alpha} & \text{production function} \\
\frac{p_{j,t}}{P_t} = \frac{\theta}{\theta-1}\frac{E_t\sum_{i=0}^{\infty} w^i\beta^i C_{t+i}^{1-1/\sigma}\varphi_{t+i}\left(\frac{P_{t+i}}{P_t}\right)^{\theta}}{E_t\sum_{i=0}^{\infty} w^i\beta^i C_{t+i}^{1-1/\sigma}\left(\frac{P_{t+i}}{P_t}\right)^{\theta-1}} & \text{price setting under sticky prices} \\
\frac{p_{j,t}}{P_t} = \frac{\theta}{1-\theta}\varphi_t \Leftrightarrow \varphi_t = \frac{1}{\mu} \text{ OR} & \text{price setting under flexible prices} \\
Y_t = C_t & \text{goods market equilibrium}
\end{cases}
$$

# The non-stochastic steady state

- At the steady state, prices are constant, so **inflation is zero** ($P_{t+k} = P_t, \forall k$).
- Through the Euler equation, imposing $P_{t+1} = P_t = P^*$ and $C_{t+1} = C_t = C^*$, we have: $i^* = \beta^{-1} - 1 = r^n$, and $\Delta_{t,t+i+1} = \beta$.

### The benchmark

The natural rate of interest $r^n$ is the rate of interest that would prevail in a fully flexible price environment (so that the aggregate demand equals the potential output).

- The real wage $\frac{W}{P}$ is given by $Z^* \equiv 1$ and $\mu$ (price setting behavior). With $C^* = Y^*$, the production function gives $C$ as function of $L$, and the labor supply gives $L$ and the real money balances $\frac{M}{P}$ though the money demand.

### A cash-less/real model

Nominal values (price level $P$ and nominal wage $W$) are undetermined.

# The log-linearization under flexible prices

1. Euler equation: $\tilde{c}_t = E_t(\tilde{c}_{t+1}) - \sigma \underbrace{E_t(r_{t+1})}_{i_t - E_t(\pi_{t+1})}$
2. Money demand: $-b\tilde{m}_t + \frac{1}{\sigma}\tilde{c}_t = \frac{1}{i^*(1+i^*)}(i_t - i^*)$ $(m \equiv \frac{M}{P})$
3. Equilibrium on the labor market: $\eta \tilde{l}_t + \frac{1}{\sigma}\tilde{c}_t = \tilde{z}_t$
4. Production function: $\tilde{y}_t = (1-\alpha)\tilde{l}_t + \tilde{z}_t$
5. Equilibrium on the goods market (without public spending): $\tilde{y}_t = \tilde{c}_t$
6. A specification for the stochastic process $\tilde{z}_t$: $\tilde{z}_t = \rho \tilde{z}_{t-1} + \text{nid}(0, \sigma_\epsilon)$

see the details of the derivation

# The log-linearization under sticky prices

1. Euler equation: $\tilde{c}_t = E_t(\tilde{c}_{t+1}) - \sigma E_t(r_{t+1})$
2. Money demand:
3. Money demand: $-b\tilde{m}_t + \frac{1}{\sigma}\tilde{c}_t = \frac{1}{i^*(1+i^*)}(i_t - i^*)$
4. Aggregate supply/pricing behavior:
   $\pi_t = \frac{(1-w)(1-w\beta)}{w}\tilde{\varphi}_t + \beta E_t(\pi_{t+1})$
5. Equilibrium on the labor market: $\eta\tilde{l}_t + \frac{1}{\sigma}\tilde{c}_t = \tilde{z}_t$
6. Production function: $\tilde{y}_t = (1-\alpha)\tilde{l}_t + \tilde{z}_t$
7. Equilibrium on the good market (without public spending):
   $\tilde{y}_t = \tilde{c}_t$
8. A specification for the stochastic process $\tilde{z}_t$:
   $\tilde{z}_t = \rho\tilde{z}_{t-1} + \mathrm{nid}(0, \sigma_\epsilon)$

see the details of the derivation

# The log-linearization under sticky prices

The model is usually expressed in terms of output gap $x_t \equiv \tilde{y}_t - \tilde{y}_t^f$, i.e. the difference between the output $y$ and the output that would prevail under flexible price $y^f$ (see Equation (18)).

1. Euler equation: $x_t = E_t(x_{t+1}) - \sigma[\tilde{i}_t - E_t(\pi_{t+1})] + \underbrace{u_t}_{E_t(\hat{y}^f_{t+1}) - \hat{y}^f_t}$
2. Money demand: $\frac{\gamma(\frac{M_t}{P_t})^{-b}}{C_t^{-\sigma}} = \frac{i_t}{1+i_t}$
3. Pricing behavior/aggregate supply:
$$\pi_t = \underbrace{(\eta + \sigma^{-1})\frac{(1-w)(1-w\beta)}{w}}_{\kappa \text{ (slope of the Phillips curve)}} x_t + \beta E_t(\pi_{t+1})$$
4. Shocks are AR(1) with autocorrelation $0 < \rho < 1$.

To close the model (i.e. to determine all endogenous variables as a function of the exogenous and predetermined variables and initial conditions), specify how $i/M$ is determined: monetary policy.

# Monetary policy: the Central Bank behavior

- With a separable utility function, the real money balances $\frac{M}{P}$ do not enter the Euler equation.
- Money is **implicit**: the Central Bank uses an interest rate rule, and the optimality condition on the money market determines the corresponding optimal quantity of money $M$.
- For instance, a contemporaneous Taylor rule (Taylor 1993) closes the model:

$$\boxed{\tilde{i}_t \equiv= i_t - i_t^* = \phi_\pi(\pi_t - \pi^T) + \phi_x(x_t)} \tag{11}$$

with $\phi_\pi$, $\phi_x \neq 0$ the coefficients of reaction of monetary policy.

- Other rules can be used (e.g. optimal rule, expectation rule, backward-looking rule, see Bullard & Mitra, JME, 2002).

# The baseline NK model: conclusions

- Current output gap $x_t$ is influenced
  - negatively by the interest rate (intertemporal substitution *via* $\sigma$);
  - positively by the **expected** output gap (expectation channel);
  - by the **real shock** $u$.
- Current inflation $\pi_t$ is influenced:
  - positively by the current output gap (demand effect);
  - positively by the **expected** inflation rate (expectation channel);
  - by the **cost-push shock** $e$ (trade-off inflation/output gap stabilization).
- Monetary policy adjusts the nominal interest rate to stabilize inflation and output gap around 0. more on inflation targeting

# The New Keynesian Phillips curve: implications

- Management of expectations (**inflation targeting** regime).

"*Not only do expectations about policy matter, but very little else matters*" (Woodford 2005, p. 3).

- A cost-push shock $e_t$ (i.e. a shock on prices or nominal wages) needs to be included to the NKPC to allow for a trade-off between stabilizing inflation and output gap:

$$\pi_t = \beta\pi^e_{t+1} + \kappa\varphi_t + e_t \tag{12}$$

- By **iterating forward**, we have $\pi_t = \kappa \sum_{i=0}^{+\infty} \varphi^e_{t+i}$. Inflation is a "jump" variable (i.e. only depends on expectations)!
- "Hybrid NKPC" to (re)introduce persistence:

$$\pi_t = \gamma\pi_{t-1} + (1-\gamma)\pi^e_{t+1} + \kappa\varphi_t + e_t \tag{13}$$

The higher $\gamma$ or the lower $\kappa$ (slope of the Phillips curve), the less power monetary policy has on inflation.

# Determinacy of the rational expectation equilibrium

We rewrite the model in matrix form:

$$\boxed{Y_t = \alpha + \Phi E_t(Y_{t+1}) + \Psi \varepsilon_t} \tag{14}$$

with $Y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}$, $\alpha = \frac{1}{d}\sigma^{-1}\phi_\pi \pi^T \begin{pmatrix} 1 \\ \kappa \end{pmatrix}$,

$\Phi = \frac{1}{d}\begin{pmatrix} 1 & \sigma^{-1}(1-\beta\phi_\pi) \\ \kappa & \kappa\sigma^{-1} + \beta(1+\sigma^{-1}\phi_x) \end{pmatrix}$,

$\Psi = \frac{1}{d}\begin{pmatrix} 1 & -\sigma^{-1}\phi_\pi \\ \kappa & 1+\sigma^{-1}\phi_x \end{pmatrix}$ and $\varepsilon_t = \begin{pmatrix} u_t \\ e_t \end{pmatrix}$

with $d = 1 + \sigma^{-1}\phi_x + \sigma^{-1}\phi_\pi\kappa \neq 0$.

# Determinacy of the rational expectation equilibrium

- Two minimum state variables ($\pi$ and $x$), no predetermined variable (for the MSV, see next slide).
- Blanchard & Kahn condition: the two eigenvalues of $\Phi$ have to lie within the unit circle for the model to have a unique rational expectation equilibrium (REE), i.e. for determinacy.
- The characteristic polynomial of $\Phi$ : $P_\Phi(\lambda) = \lambda^2 + a_1\lambda + a_0$ $\left(a_0 = \frac{\beta}{d} \quad \text{and} \quad a_1 = -\frac{1}{d}\left[1 + \kappa\sigma^{-1} + \beta(1 + \sigma^{-1}\phi_\pi)\right]\right)$.
  So we need:

$$\begin{cases} |a_0| < 1 \\ |a_1| < 1 + a_0 \end{cases} \Leftrightarrow \begin{cases} \text{true as } \beta \in ]0,1[ \\ \boxed{\phi_\pi + \dfrac{1-\beta}{\kappa}\phi_x > 1} \end{cases} \Rightarrow \text{Taylor principle}$$

# Computing the Rational expectation equilibrium

**The Minimum State Variable (MSV) solution** reads as:

$$Y_t = \bar{a} + \bar{c}\varepsilon_t$$

With the method of undetermined coefficients:

$$\bar{Y}_t = \alpha + \Phi \bar{Y}_t + \Psi \varepsilon_t$$

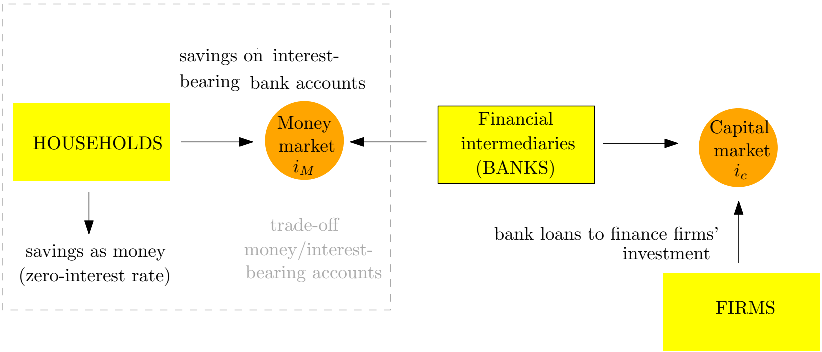
$$\Leftrightarrow \bar{Y}_t = (I - \Phi)^{-1}\alpha + (I - \Phi)^{-1}\Psi\varepsilon_t$$

which gives:

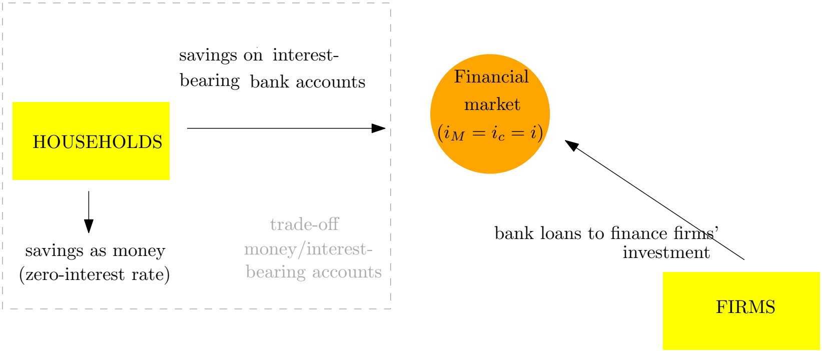
$$\Leftrightarrow \boxed{\bar{a} = (I - \Phi)^{-1}\alpha} \text{ and } \boxed{\bar{c} = (I - \Phi)^{-1}\Psi}.$$

▸ more on the computation of the REE

# In reality ...




$\Rightarrow$ Banks serve as financial intermediaries between households' savings and firms' financial needs.



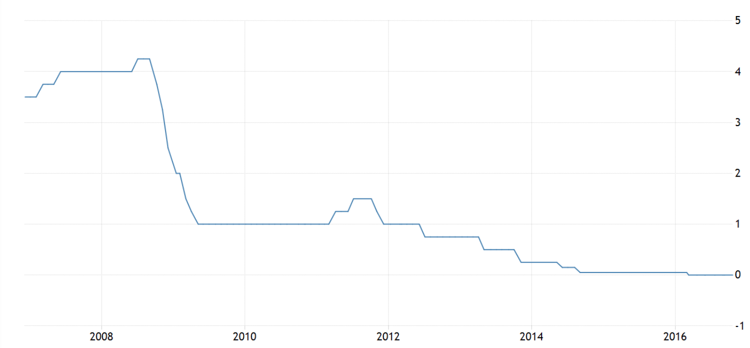

$\Rightarrow$ Only two agents: firms and households (+ policy makers).

# Extension of the baseline NK models

- A more complicated production sector (intermediary whole-sale sector that produces the capital goods).
- Search and matching model of unemployment. $w_t^* \equiv max\left\{(H_t)^{\eta_t}(J_t)^{1-\eta_t}\right\}$, $0 < \eta < 1$ is the workers' bargaining power, $H_t$ households' outside option, and $J_t$ the firms' marginal value of a job.
- "Rule-of-thumb"/Non Ricardian and optimizing consumers: type "r" maximizes a static problem $U(C_t, L_t)$, under intra-period budget constraint $P_t C_t^r = W_t L_t^r \rightarrow C_t^r \propto \frac{W_t}{P_t}$.
- Habit persistence. $u(C) = \frac{(c_t - \nu h_t)^{1-\sigma}}{1-\sigma}$ with $h_t = (1-\delta-)h_{t-1} = \sum_{j=1}^{\infty}(1-\delta)^{j-1}c_{t-j}$.
- Financial frictions.
- Zero-lower bound of the nominal interest rate and unconventional monetary policy.

# Monetary policy reactions: Euro area

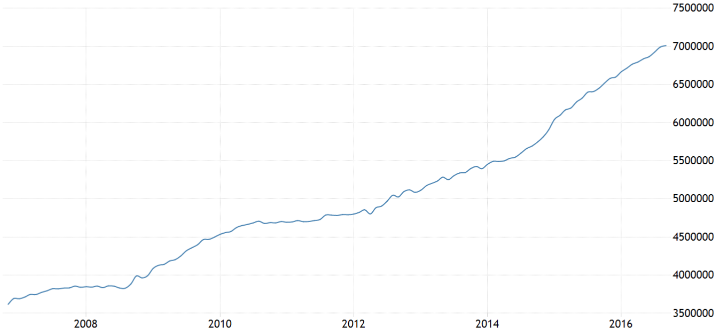
## Evolution of the benchmark interest rate in the Euro area




Source: ECB.

# Policy reactions, Euro area

## Evolution of the money stock (M1), Euro area, 2007-2016




Source: ECB.

# How to manage inflation expectations?

→ If drop in $\pi^e$ is strong enough to reach the ZLB, $\pi^e$ is the only driver of $r$, but $\pi^e$ is **self-fulfilling**!

→ Inflation is positively related to developments in aggregate demand → during a crisis, inflation expectations should go down!

→ the need for **credible commitment to deliver future higher inflation**.

- Increasing the money supply but liquidity trap.
- Decreasing longer-run interest rate, the so-called spreads.
- **Forward guidance** (statement delivering about future policy).
- Institutional frameworks (inflation targeting).
- Changing monetary policy objectives.

# Lowering interest rate differentials in a liquidity trap

- In practice, households' **deposits rate** $i_M$ and on **firms' loans** rate $i_c$ differ.
- A **financial crisis** affects the agents' **confidence** and **risk perception** in the money and capital markets: households may loose confidence in banks' solvency, and banks may loose confidence in firms' ability to pay back loans.
- Because of **risk** (and horizon) of the loan, deposit and lending rates may remain positive, even in a liquidity trap: **risk premium**.

→ **Financial accelerator** and credit market disruption: aggregate demand effect.

# From 2014: non-conventional monetary policy

**Why?**: Weak recovery + fall in oil price → inflation threat away, deflation threat considered as serious.

**What?**

- **Targeted longer-term refinancing operations** (TLTROs) from 2014, update in 2016: provide funds to banks for lending to the private sector.
- **Quantitative easing**:
  - January 2015: 60 b. euro/month for, at least, 19 months, incl. at least 50 b. euro/month in public debt (14% of public debt).
  - March/July/Oct. 2016: 80 b. euro/month until "*the end of March 2017, or beyond, if necessary, and in any case until it sees a sustained adjustment in the path of inflation consistent with its inflation aim*".

# From 2014: non-conventional monetary policy

**What?**

- **Forward-guidance**: "*The Governing Council continues to expect the key ECB interest rates to remain at present or lower levels for an extended period of time, and well past the horizon of the net asset purchases*", July, unchanged Oct. 2016.

**What for?**

- Weaken euro vs. dollar: competitiveness.
- Provide liquidity to the banking sector and decrease longer-term lending rates: investment.
- Psychological impact: confidence.

# Non-Rational Expectations and Learning

- **Underlying assumption** : The representative agent is supposed to know the form of the REE, and the values of the model's coefficients.
- **Question**: is that realistic ?
- **Alternative (more plausible) assumptions**: **the representative agent tries to learn how to form rational expectations**.
- **Question**: **Does the representative agent learn to have rational expectations?** Do the expectations of the learning agent converge towards their REE values?

# Least-square/adaptive learning

"When implemented numerically or econometrically, rational expectations models impute much more knowledge to the agents within the model [...] than is possessed by an econometrician, who faces estimation and inference problems that the agents in the model have somehow solved."

Thomas Sargent (1993)

# Least-squares learning in the NK model

References: Sargent (1993), Evans & Honkapohja (2001)

At time $t$, the representative agent:

1. has the information set $(1,\ \varepsilon_t)'$, for $t = 0, ..., t-1, t$.
2. estimates the **Perceived Law of Motion (PLM)**:

$$Y_t = a_t + c_t \varepsilon_t \tag{15}$$

3. updates estimates $\hat{\beta}_t = (a_t, c_t)'$ of $\bar{a}$ and $\bar{c}$ every period (reruns the regression), as the last period's observation $(1,\ \varepsilon_t)'$ becomes observable (recursive least squares):

$$\hat{\beta}_t = \hat{\beta}_{t-1} + \gamma_t R_t^{-1}(1,\ \varepsilon_{t-1})' \left( Y_{t-1} - \hat{\beta}'_{t-1}(1,\ \varepsilon_{t-1})' \right)$$

4. forms his expectations using his PLM:

$$E_t(Y_{t+1}) = a_t + c_t \rho \varepsilon_t \Rightarrow \begin{cases} E_t(x_{t+1}) = a_{1,t} + c_{1,t} \rho \varepsilon_t \\ E_t(\pi_{t+1}) = a_{2,t} + c_{2,t} \rho \varepsilon_t \end{cases}$$

# Least-squares learning in the NK model

**Questions**:

- Do the estimates $(a_t, c_t)$ converge towards the REE values $(\bar{a}, \bar{c})$ when $t \to \infty$?
  $\Rightarrow$ **Concept of E-stability/learnability**.

- If yes, under which conditions? Does the convergence depend on monetary policy (i.e. $\phi_\pi$ and $\phi_x$)?

- In particular, is the Taylor principle enough to ensure convergence?

  $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$

$\Rightarrow$ How to analyze this question? very similarly as before!

# Least-squares learning in the NK model

1. Recall the **Perceived** Law of Motion (PLM):

$$Y_t = a_t + c_t \varepsilon_t$$

and the resulting expectations:

$$E_t(Y_{t+1}) = a_t + c_t \rho \varepsilon_t$$

2. Plug the expectations into the reduced-form model and get the resulting **Actual** Law of Motion (ALM):

$$Y_t = \alpha + \Phi a_t + (\Phi c_t \cdot \rho + \Psi) \varepsilon_t$$

# Least-squares learning in the NK model

3. Define the **T-map**:

$$T(a_t, c_t) = (\alpha + \Phi a_t, \Phi c_t \cdot \rho + \Psi)$$

→ Fixed point of the T-map corresponds to the REE!

4. So, write down the dynamical system:

$$T(a_t, c_t) - (a_t, c_t) \Rightarrow (\alpha + \Phi a_t, \Phi c_t \cdot \rho + \Psi) - (a_t, c_t) = 0$$

→ The REE $(\bar{a}, \bar{c})$ is E-stable if the fixed point $(\bar{a}, \bar{c})$ of this system is locally asymptotically stable.

# Least-squares learning in the NK model

## E-stability condition

Reference: Bullard & Mitra (2002)

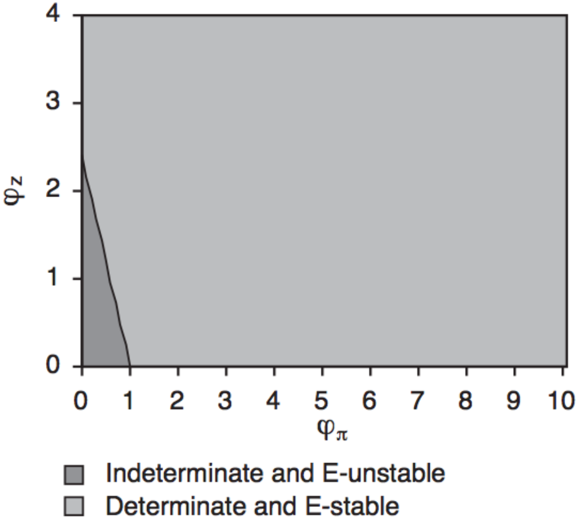
$$T(a, c) - (a, c) = (\alpha + \Phi\bar{a}, \Phi\rho\bar{c} + \Psi) - (\bar{a}, \bar{c}) = (0, 0)$$

Both eigenvalues of $\Phi$ and $\rho\Phi$ have to be lower than one (in absolute value) $\Rightarrow$ enough to prove for $\Phi$ (as $|\rho| < 1$).

The REE is E-stable if and only if $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$ (see above) $\Rightarrow$ the Taylor principle has to hold.

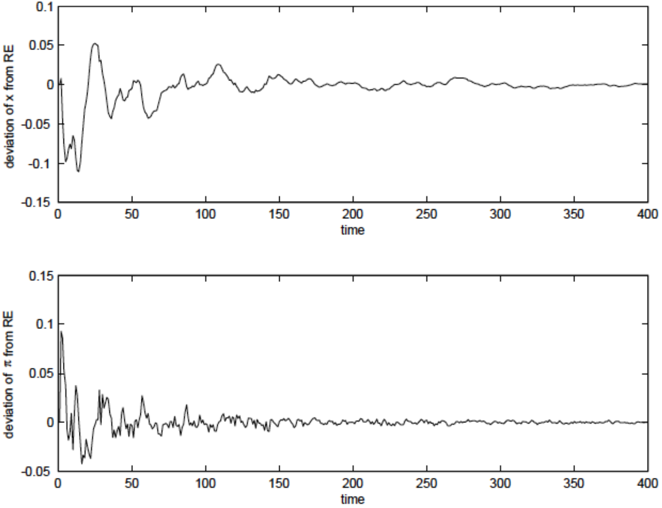
# Least-squares learning in the NK model

## Recommendation in terms of monetary policy (Bullard & Mitra (2002)



# Least-squares learning in the NK model

## Under the Taylor principle



# Remarks on least-squares learning in the NK model

- Agents do not know the values of the REE coefficients of matrix $\bar{a}$ and $\bar{c}$ but are supposed to know the underlying economic model.
- There are some extensions in which the PLM is mispecified (e.g. include lagged values of $Y$).
- They are also assumed to know the true value of $\rho$.
- Still a representative agent framework, no heterogeneity.
- There are some extensions with two agents and two different PLMs (e.g. Honkapohja & Mitra 2006).

  $\rightarrow$ What if we assume a population of heterogeneous agents, each of them with different estimated $a$ and $c$?

# Social learning in the NK model

## Genetic algorithm (GA)-based population learning

*Reference*: Arifovic *et al.* (2013)

- A population of $N$ agents, $i = 1, ..., N$, each of them forecasting output gap and inflation according to **his own PLM** (**agents are heterogeneous**):

$$Y^e_{i,t+1} = a_{i,t} + c_{i,t}\rho\varepsilon_t \Rightarrow \begin{cases} x^e_{i,t+1} = a_{1,i,t} + c_{1,i,t}\rho\varepsilon_t \\ \pi^e_{i,t+1} = a_{2,i,t} + c_{2,i,t}\rho\varepsilon_t \end{cases}$$

- Define the 4 coefficients $(a_{1,i,t}, a_{2,i,t}, c_{1,i,t}, c_{2,i,t})$ as the strategies of agent $i$ in time $t$.
- At each period $t$, we have a population of $N$ strategies $(a_{1,i,t}, a_{2,i,t}, c_{1,i,t}, c_{2,i,t})$, $i \in \{1, ..., N\}$.

# Social learning in the NK model

## Interesting questions

- How to model agents' **interactions**?
- How does the population of strategies evolve over time?
- Do agents **coordinate** on the REE strategy $(\bar{a_1}, \bar{a_2}, \bar{c_1}, \bar{c_2})$ after a high number of iterations?
- If yes, at which conditions? are the implications in terms of monetary policy rules (especially the Taylor principle) alter when considering heterogeneous and interacting agents?

# Evolutionary learning: sequence of events

1. Initialize the strategy $(a_{1,i,0}, a_{2,i,0}, c_{1,i,0}, c_{2,i,0})$ for the $N$ agents $i$.
2. Compute the average expectations across the population:
$$x^e_{t+1} = \frac{1}{N}\sum_{i=1}^{N} x^e_{i,t+1}$$
$$\pi^e_{t+1} = \frac{1}{N}\sum_{i=1}^{N} \pi^e_{i,t+1}$$
3. Plug $x^e_{t+1}$ and $\pi^e_{t+1}$ back into the reduced form model $Y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} \alpha + \Phi E_t(Y_{t+1}) + \Psi \varepsilon_t$.
4. Compute the forecast error (fitness) of each forecast strategy $i$:
$$F_{i,t} \propto -(\pi^e_{i,t} - \pi_t)^2 - (x^e_{i,t} - x_t)^2$$
5. Apply a **genetic algorithm** on the population of $N$ strategies.

# Modeling social learning: genetic algorithms

- Two main principles:
  - **Exploitation/selection**: duplicate the most successful strategies in the next period's population and discard the less accurate ones
    $\Rightarrow$ *survival of the fittest*.
  - **Exploration/diversity**: introduce new strategies in the next period's population to discover potential better strategies than existing ones.
    $\Rightarrow$ diversity at the source of better economic performances.
- Three operators:
  - Imitation
  - Cross-over
  - Mutation

# Imitation/reproduction

## Selection procedure of the best strategies to be imitated

With a probability $P_{imit}$ each period:

- either *tournament selection* – select a subset $n$ of the $N$ strategies (random draw) and adopt the best strategies among the $n$ of the subset.
- or *roulette-wheel selection* – a strategy is imitated with a probability which is proportional to its relative fitness $F_{i,t}$ in the population:

$$\frac{F_{i,t}}{\sum_{i=1}^{N} F_{i,t}}$$

*biased roulette wheel* where each strategy is allocated a slot sized in proportion to its fitness. The number of spins of the wheel is equal to $N$, the number of strategies in the population.

# Cross-over

## models communication between agents and exchange of ideas

With a probability $P_{co}$ each period:

1. Selection of two strategies (random draw) to be the two *mates*:

$$(a_{1,1,t}, a_{2,1,t}, c_{1,1,t}, c_{2,1,t}) \text{ and } (a_{1,2,t}, a_{2,2,t}, c_{1,2,t}, c_{2,2,t})$$

2. Randomly select a number between 1 and the number of components of the strategies (here 4), e.g. 2.
3. Cross-over the two mates to obtain two new strategies to replace the two mates:

$$(a_{1,1,t}, a_{2,1,t}, |c_{1,1,t}, c_{2,1,t})$$
$$(a_{1,2,t}, a_{2,2,t}, |c_{1,2,t}, c_{2,2,t})$$

become

$$(a_{1,1,t}, a_{2,1,t}, |c_{1,2,t}, c_{2,2,t})$$
$$(a_{1,2,t}, a_{2,2,t}, |c_{1,1,t}, c_{2,1,t})$$

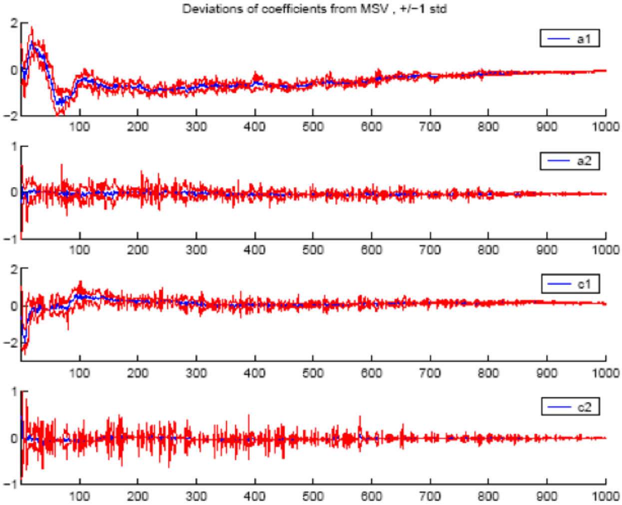
# Social learning *via* a genetic algorithm

## models the introduction of new ideas which can be good innovations or mistakes

With a given probability $P_{mut}$ each period, each component of each strategy is changed (random draw in the strategy domain).

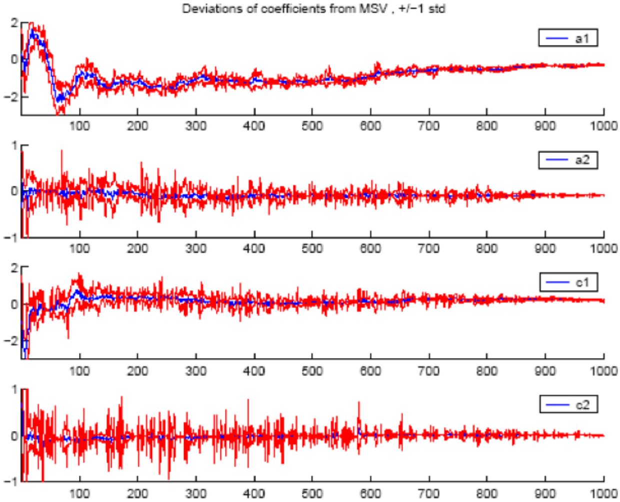
# Social learning in the NK model : results (1)

## Convergence under the Taylor principle



# Social learning in the NK model : results (2)

## Convergence without the Taylor principle



# Remarks on social learning in the NK model

- Main difference with LS learning: the need for complying with the Taylor principle is qualified, as convergence is obtained for combinations of policy parameters for which $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$ does not hold.
- ... but this is only true if we consider that agents have the right representation of the econometric model, i.e. $Y(\text{intercept}, \varepsilon_t)$.
- This is a strong assumption, even econometricians do not know beforehand the true specification of economic models.
- What if agents use simpler predictors (heuristics) ?

# Heterogeneous expectations in the NK framework

## Heuristic switching model

References: Branch & McGough (2009)

- The simple NK model:

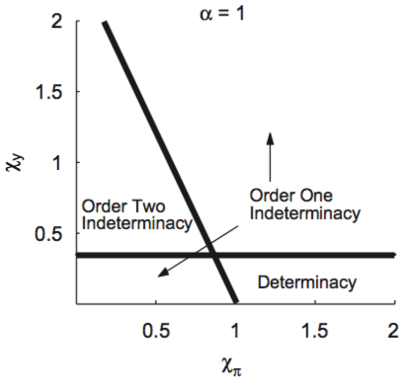
$$x_t = \hat{E}_t(x_{t+1}) - \sigma^{-1}(i_t - \hat{E}_t(\pi_{t+1}))$$
$$i_t = \phi_\pi E_t(\pi_{t+1}) + \phi_x E_t(x_{t+1})$$
$$\pi_t = \beta \hat{E}_t(\pi_{t+1}) + \kappa x_t$$

- Specification of heterogeneous expectations ($z = \pi, x$):

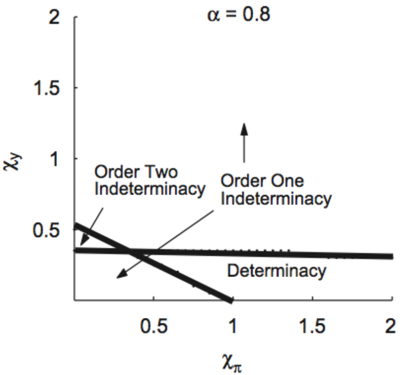
$$\left.\begin{array}{r} \alpha \text{ agents are rational} \\ (1-\alpha) \text{ agents are adaptive} \end{array}\right\} \hat{E}_t(z_{t+1}) = \alpha E_t(z_{t+1}) + (1-\alpha)\theta z_{t-1}$$

$\theta = 1 \Leftrightarrow$ naive expectations.
$\theta < 1 \Leftrightarrow$ adaptive expectations.
$\theta > 1 \Leftrightarrow$ extrapolative expectations.

# What happens to the Taylor principle? $\theta = 0.95$



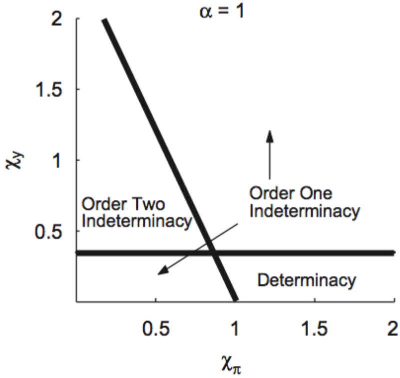

(a) $\alpha = 1$ (RE)



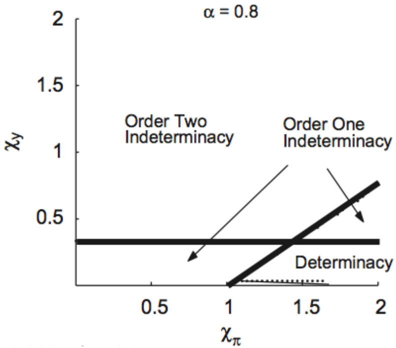

(b) $\alpha = 0.8$

→ Adaptive agents ($\theta = 0.95 < 1$) stabilize the economy !

# What happens to the Taylor principle? $\theta = 1.05$




(c) $\alpha = 1$ (RE)




(d) $\alpha = 0.8$

→ Extrapolative agents ($\theta = 1.05 > 1$) destabilize the economy !

# An heuristic-switching model

References: Branch & McGough (2010)

- Two predictors for inflation $\pi$:

$$\left.\begin{aligned} E_t^1(\pi_{t+1}) &= E_t(\pi_{t+1}) \\ E_t^2(\pi_{t+1}) &= \theta\pi_{t-1} \end{aligned}\right\} \hat{E}_t(\pi_{t+1}) = n_t E_t(\pi_{t+1}) + (1 - n_t)\theta\pi_{t-1}$$

- Fitness measure : forecast accuracy
  - Predictor 1 (rational): $U_t^1 = -(\pi_t - E_t(\pi_{t+1}))^2 - C$
  - Predictor 2 (adaptive): $U_t^2 = -(\pi_t - \theta\pi_{t-1})^2$

  $\rightarrow$ $C$ is the relative cost of rationality.
- Discrete choice model (predictors dynamics):

$$n_t = \frac{\exp(\beta U_t^1)}{(\exp(\beta U_t^1) + \exp(\beta U_t^2))} \qquad (16)$$

$\rightarrow$ intensity of choice: $\beta < \infty \Leftrightarrow$ forecasting error/non-RE

# Resulting non-linear dynamical system

$$x_t = \hat{E}_t(x_{t+1}) - \sigma^{-1}(i_t - \hat{E}_t(\pi_{t+1}))$$

$$i_t = \phi_\pi E_t(\pi_{t+1}) + \phi_x E_t(x_{t+1})$$

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t$$

$$\hat{E}_t(\pi_{t+1}) = n_t E_t(\pi_{t+1}) + (1 - n_t)\theta\pi_{t-1}$$

$$\hat{E}_t(x_{t+1}) = n_t E_t(x_{t+1}) + (1 - n_t)\theta x_{t-1}$$

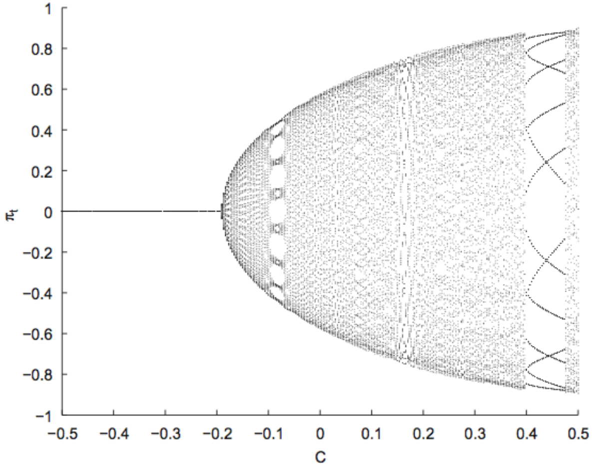
$$U_t^1 = -(\pi_t - E_t(\pi_{t+1}))^2 - C$$

$$U_t^2 = -(\pi_t - \theta\pi_{t-1})^2$$

$$n_t = \frac{\exp(\beta U_t^1)}{(\exp(\beta U_t^1) + \exp(\beta U_t^2))}$$

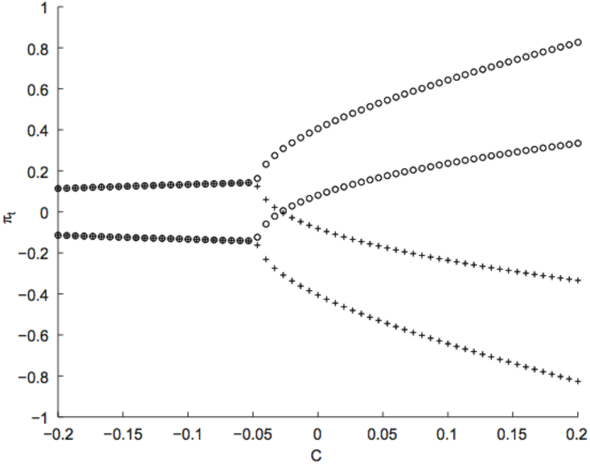
# Bifurcation diagram under the Taylor principle

$\theta = 1.05$, $\phi_\pi = 1.1$ and $\phi_x = 0.35$



# Bifurcation diagram without the Taylor principle

$\theta = 0.7$, $\phi_\pi = 0.3$ and $\phi_x = 0.5$



# Intuitions behind complex dynamics

Reference: Brock & Hommes (1997)

- Key: adaptive and rational predictors return distinct forecasts out of steady state → predictor cost *versus* forecast accuracy.
- Start close to the steady state, both yield identical forecast → when $C$ is large, most agents use the adaptive predictor.
- Consequently, inflation and output diverge from the steady state, and start oscillating → adaptive predictor less accurate.
- Far enough from the steady state, it becomes profitable to buy the rational expectations forecast → the proportion of rational agents rises, causing the steady state to become attracting and drawing the economy toward it.
- Close to the steady state, the relative accuracy of the rational predictor falls, and agents begin switching to the (cheaper) adaptive predictor → the steady state repels the economy and the process repeats itself.

# Remarks on heuristics in the NK model

- Heterogeneity can strongly impact a NK model: with a natural evolution of $n$, a model that is determinate under RE may yield complex dynamics.
- Whether heterogeneity is stabilizing or destabilizing depends on the distribution and nature of the heterogeneity.
- What specification of heterogeneous expectations is consistent with economic data?
- How to measure expectations? How to estimate models of expectations formation?
- Experimental economics: learning to forecast experiments.

# Learning to forecast experiments in the NK model

References: Assenza et al. (2012)

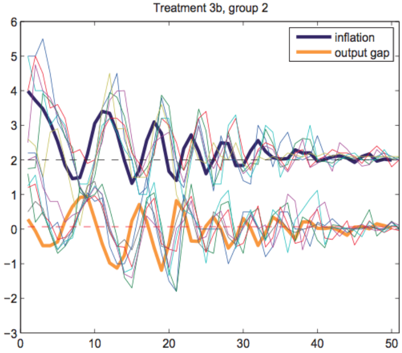
- The simple NK model:

$$
\begin{aligned}
x_t &= E_t(x_{t+1}) - \sigma^{-1}(i_t - E_t(\pi_{t+1}) - r_t^n) \\
i_t &= \phi_\pi \pi_t \\
\pi_t &= \beta E_t(\pi_{t+1}) + \kappa x_t + u_t
\end{aligned}
$$

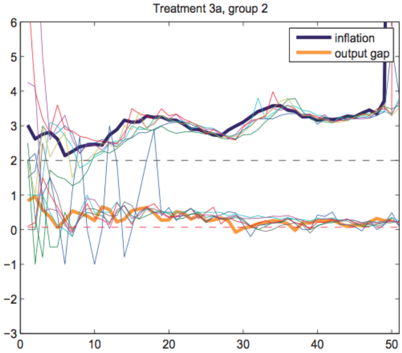
  $r_t^n$ and $u_t$ i.d.d. shocks, $\phi_x = 0$
  $\Rightarrow$ the Taylor principle reduces to $\phi_\pi > 1$.
- Subjects do not know the equations of the underlying economic model.
- Subjects have to forecast output gap and inflation with $\phi = 1$ or $= 1.5$, observing their past forecasts and the past values of the aggregate variables.
- Subjects are paid according to their forecast accuracy.

# Experiments in the NK model

## Learning to forecast **both** inflation and output gap expectations



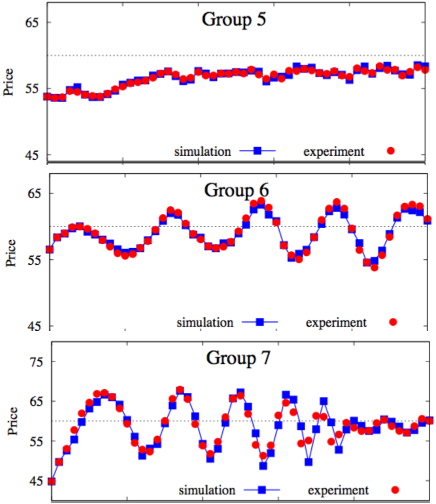

(e) Taylor principle




(f) non Taylor principle

# Fitting models to experimental data

Reference: Hommes (2011)



# Main limitations of NK models

Some of limitations of RBC models still apply to NK models:

- No, or limited **heterogeneity** → the representative agent rules out the possibility of coordination failures.
- No involuntary unemployment, aggregate demand effects are very limited.
- "Fitting is not explaining" → fluctuations and persistence are exogenous: prediction?
- Cognitive abilities of agents almost unlimited (infinite horizon, RE, etc.).

# Macro-models: a compared approach

| | Keynesian models (e.g. $IS - LM$) | DSGE models |
|---|---|---|
| Microeconomic foundations | missing | representative agent |
| Agents' behaviour | not modelled | maximisation, rational expectations |
| Macroeconomic relationships | postulated and estimated | mapping with individual behaviour |
| Source of fluctuations | exogenous shocks | exogenous shocks |

# Macro-models: a compared approach

| | Keynesian models (e.g. $IS - LM$) | DSGE models | Agent-based macro models |
|---|---|---|---|
| Microeconomic foundations | missing | representative agent | interacting and heterogeneous agents |
| Agents' behaviour | not modelled | maximisation, rational expectations | procedural rationality, local information |
| Macroeconomic relationships | postulated and estimated | mapping with individual behaviour | emerging from individual and local interactions |
| Source of fluctuations | exogenous shocks | exogenous shocks | endogeneous: coordination issues |

Thank you very much for your attention

# Computing the discount rate

- Let us define $R_t \equiv \frac{1+i_t}{1+\pi_{t+1}}$, as the real interest rate between periods $t$ and $t+1$ ($\pi_{t+1} \equiv \frac{P_{t+1}-P_t}{P_t}$ the inflation rate, $i_t$ the nominal interest rate).
- Notice that $R_t = \frac{P_t}{P_{t+1}}(1+i_t) \Leftrightarrow \frac{1}{R_t} = \frac{P_{t+1}}{P_t(1+i_t)}$.
- To discount any value $X_{t+n}$ in period $t$:
  $\forall n,\ \frac{1}{R_t} \times \cdots \times \frac{1}{R_{t+n}} X_{t+n} = \prod_{k=0}^{n} \frac{1}{R_{t+k}} X_{t+n}$
  $\Leftrightarrow \frac{P_{t+1}}{P_t} \frac{1}{(1+i_t)} \times \frac{P_{t+2}}{P_{t+1}} \frac{1}{(1+i_{t+1})} \times \cdots \times \frac{P_{t+n+1}}{P_{t+n}} \frac{1}{(1+i_{t+n})} X_{t+n}$
  $\Leftrightarrow \frac{P_{t+n+1}}{P_t} \prod_{k=0}^{n} \frac{1}{(1+i_{t+k})} X_{t+n}$
- From Euler equation (3), we have:
  $E_t\left(\frac{1}{1+i_{t+k}} \frac{P_{t+k+1}}{P_{t+k}}\right) = \beta E_t\left(\left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-1/\sigma}\right)$
- Computing the discount factor between $t$ and $t+n$:
  $\prod_{k=0}^{n} E_t\left(\frac{1}{1+i_{t+k}} \frac{P_{t+k+1}}{P_{t+k}}\right) = \prod_{k=0}^{n}\left[\beta E_t\left(\left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-1/\sigma}\right)\right] =$
  $\beta^{n+1} \frac{C_{t+n+1}}{C_t}$ back

# More on the log-linearization under flexible prices

- Market clearing on the labor market gives:

$$\frac{W_t}{P_t} = \frac{Z_t}{\mu} = \frac{N_t^{\eta}}{C_t^{-1/\sigma}} \tag{17}$$

- Log-linearizing (17) gives: $\eta \tilde{n}_t^f + \sigma^{-1} \tilde{c}_t^f = \tilde{z}_t$;
- and the output under flexible prices is:

$$\boxed{\tilde{y}_t^f = \frac{1+\eta}{\sigma^{-1}+\eta} \tilde{z}_t} \tag{18}$$

where market clearing in the goods market without government expenditures implies $\tilde{c} = \tilde{y}$

# More on the log-linearization under sticky prices

- We define $Q_t \equiv \frac{P_t^*}{P_t}$, the relative price of firms that can adjust their price in $t$ to the aggregate price index.
- We have $Q^* = 1$ and $\tilde{P}_t^* = \tilde{q}_t + \tilde{P}_t$.
- By definition, inflation is $\pi_t \equiv \ln(P_t) - \ln(P_{t-1}) = \tilde{P}_t - \tilde{P}_{t-1}$.
- Log-linearizing Equation (10) gives:

$$\tilde{P}_t = (1-w)\tilde{P}_t^* + w\tilde{P}_{t-1} \Leftrightarrow \pi_t = \tilde{P}_t - \tilde{P}_{t-1} = (1-w)(\tilde{P}_t^* - \tilde{P}_{t-1})$$

$$\Leftrightarrow (1-w)\tilde{q}_t - w\pi_t = 0 \quad \Leftrightarrow \quad \boxed{\tilde{q}_t = \frac{w}{1-w}\pi_t}$$

- which gives:

$$\tilde{P}_t^* = (1-w\beta)\sum_{i=0}^{\infty}(w\beta)^i E_t(\tilde{\varphi}_{t+i} + \tilde{p}_{t+i}) \tag{19}$$

with $\tilde{\varphi} + \tilde{p}$ the nominal marginal cost.

# More on the log-linearization under sticky prices

- From (19), we have:

$$\tilde{q}_t = (1 - w\beta)\tilde{\varphi}_t + w\beta[E_t(\tilde{q}_{t+1}) + E_t(\pi_{t+1})]$$

- and we obtain:

$$\boxed{\pi_t = \frac{(1-w)(1-w\beta)}{w}\tilde{\varphi}_t + \beta E_t(\pi_{t+1})}$$

- Expressed as a function of the output gap $x_t$, we can show that $\varphi_t = (\eta + \sigma^{-1})x_t$, and we have the New Keynesian Phillips Curve:

$$\boxed{\pi_t = \underbrace{(\eta + \sigma^{-1})\frac{(1-w)(1-w\beta)}{w}}_{\kappa} x_t + \beta E_t(\pi_{t+1})} \tag{20}$$

The slope $\kappa$ is decreasing in $w$ and $\beta$, i.e. less adjustment possibilities or more weight on future profits give more weight to future marginal costs)

# More on the log-linearization of the Euler equation

Log-linearizing the Euler equation (3), we have:

$$\tilde{c}_t = E_t(\tilde{c}_{t+1}) - \sigma \underbrace{\left[\tilde{i}_t - E_t(\pi_{t+1})\right]}_{\text{expected real interest rate } E_t(r_{t+1})} \tag{21}$$

We have to express the Euler equation as a function of the output gap $x_t \equiv \tilde{y}_t - y_t^f$.

$$C_t = Y_t \quad \Leftrightarrow \quad \tilde{c}_t = \tilde{y}_t = x_t + \tilde{y}_t^f$$

$$\boxed{x_t = E_t(x_{t+1}) - \sigma\left[\tilde{i}_t - E_t(\pi_{t+1})\right] + \underbrace{u_t}_{E_t(\hat{y}_{t+1}^f) - \hat{y}_t^f}} \tag{22}$$

$u_t$ is a real shock on the output gap due to a technology shock.

$u_t$ would include the fiscal policy shock if the model includes public expenditures, denoted by $g$:

$\frac{\bar{Y}}{\bar{C}}\left[E_t(\tilde{y}_{t+1}^f) - \tilde{y}_t^f\right] + \frac{\bar{G}}{\bar{C}}\left[\tilde{g}_t - E_t(\tilde{g}_{t+1})\right]$ back

# Inflation targeting regimes

- Initiated in 1989 in New Zealand, today 23 countries, including 17 emerging and Eastern European countries (e.g. Canada, UK, Brazil, Ghana, Guatemala, Philippines).
- Main characteristics (Orphanides & Williams 2007):
  - Explicit inflation target and **priority** to price stabilization in the CB's mandate (e.g. $8 \pm 2\%$ in Ghana, $4 \pm 1\%$ in Indonesia, $1 - 3\%$ in Israel, 2.5% in Norway).
  - Essential role of **expectations**: inflation forecast targeting.
  - Clear **communication** strategy with the public: **transparency** leads to accountability and predictability of monetary policy decisions: publication of minutes, press conferences, speeches, publication of forecasts, **forward guidance** though future path of interest rates.
- Flexible inflation targeting: **secondary objective in terms of GDP** growth/unemployment.

# Targeting (low) inflation: why?

- After the stagflation of the 70s, no long-run trade-off between unemployment and inflation.
- Inflation is **costly** in terms of economic activity:
  - **Higher inflation is also more volatile**: inflation increases the uncertainty regarding the future rates of return of **investment**.
  - Along the same lines, inflation decreases the incentive to **save**: hedging (into precious metals or land) and capital flight (into foreign assets).
  - Inflation magnifies the microeconomic distortions created by the tax system.
  - Inflation diverts productive capacities into "unproductive" activities.
- Inflation is (partly) a **self-fulfilling process**: hard to stabilize.
- So is deflation (self-fulfilling process, Fischer effect on debt and interest rates)

# Inflation targeting regimes

## Basic requirements

- Central bank **independence** (instrument, but also goal to some extent).
- **No targeting of the nominal exchange rate**: CB's autonomy by shift toward more flexible exchange-rate regimes.
- A sufficiently strong financial system to ensure the pass-though from changes in policy rates to lending rates → "**financial repression**" (McKinnon, 1973): keeping interest rates very low by governmental interventions to limit the costs of public financing instruments (e.g. imposing large reserve and liquidity requirements on banks, ceilings on lending rates, political pressure on state-owned banks, limiting the degree of competition on the credit market).
- Increased **transparency, accountability, information** (e.g. data on prices and real sector developments, availability of reliable procedures for forecasting inflation). back

# More on the computation of the rational expectation equilibrium (REE)

**Rational expectations – definition**: $E_t(x_{t+1})$ and $E_t(\pi_{t+1})$ are the conditional mathematical expectations of $\pi$ and $x$
→ Stochastic shocks/uncertainty.

→ Let's go back to the reduced form of the stochastic model:

$$Y_t = \alpha + \Phi E_t(Y_{t+1}) + \Psi \varepsilon_t \tag{23}$$

→ How to find the rational expectations solution?
→ Under which conditions is this rational expectations equilibrium (REE) unique? **Determinacy**.

# Computation of the rational expectation equilibrium

1. Start with the reduced form of the model:

$$Y_t = \alpha + \Phi E_t(Y_{t+1}) + \Psi \varepsilon_t \tag{24}$$

Now, $Y$ not only depends on expectations but on the shock $\varepsilon_t$.

2. Consequently, the *Minimum State Variables (MSV) Solution* is:

$$Y_t = \bar{a} + \bar{c}\varepsilon_t \tag{25}$$

$\bar{a}$ and $\bar{c}$ are $2 \times 1$ vectors, whose coefficients have to be determined to find the REE.

$\Rightarrow$ This MSV solution is unique, i.e. the NK model is **determinate** iif $\phi_\pi + \frac{1-\beta}{\kappa}\phi_x > 1$ (Taylor principle).

# Computation of the rational expectation equilibrium

3. Write the law of motion for $Y_{t+1}$:

$$Y_{t+1} = \bar{a} + \bar{c} \cdot \varepsilon_{t+1} = \bar{a} + \bar{c} \cdot (\rho \varepsilon_t + e_t) \tag{26}$$

4. Apply the mathematical expectation operator :

$$E_t(Y_{t+1}) = E_t(\bar{a} + \bar{c} \cdot (\rho r_t^n + e_t)) = \bar{a} + \bar{c} \cdot \rho \varepsilon_t \tag{27}$$

(assuming $\rho$ is known and $\varepsilon_t$ is observable in $t$).

5. Substitute it back into the reduced form of the model:

$$\begin{aligned} Y_t = \alpha + \Phi E_t(Y_{t+1}) + \Psi \varepsilon_t &\Rightarrow Y_t = \alpha + \Phi(\bar{a} + \bar{c} \cdot \rho \varepsilon_t) + \Psi \varepsilon_t \\ &\Rightarrow Y_t = \alpha + \Phi \bar{a} + (\Phi \bar{c} \cdot \rho + \Psi) \varepsilon_t \end{aligned}$$

# Computation of the rational expectation equilibrium

5. We now have (from the expectations):

$$Y_t = \alpha + \Phi \bar{a} + (\Phi \bar{c} \cdot \rho + \Psi)\varepsilon_t$$

and the REE (by definition):

$$Y_t = \bar{a} + \bar{c}\varepsilon_t$$

6. Solve for the coefficients of matrix $\bar{a}$ and $\bar{c}$ (undetermined coefficients method):

$$\begin{cases} \alpha + \Phi \bar{a} = \bar{a} \\ (\Phi \bar{c} \cdot \rho + \Psi) = \bar{c} \end{cases} \Leftrightarrow \begin{cases} \boxed{\bar{a} = 0} \text{ as } \alpha = 0 \text{ and } \Phi \neq I_2 \\ \boxed{\bar{c} = (I_2 - \rho\Phi)^{-1}\Psi} \end{cases}$$

back