Advanced Macroeconomics
–
DSGE modeling – First generation of models (Real Business Cycle models)

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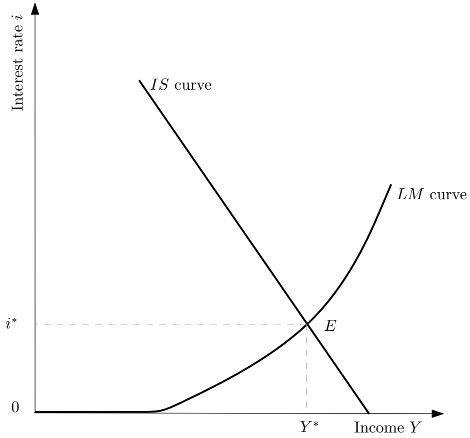
Remember the $IS - LM$ model



IS curve: $Y = C + I + G$

$$= C_0 + c(Y - T) + I_0 - b \cdot i + G$$

$$\Rightarrow i = \frac{C_0 + I_0}{b} + \frac{G}{b} - \frac{c}{b} T + \underbrace{\frac{(c-1)}{b}}_{<0} Y$$

LM curve: $L^s = L^d$

$$= l_0 + l_1 \cdot Y - l_2 \cdot i$$

$$\Rightarrow i = \frac{l_0}{l_2} + \underbrace{\frac{l_1}{l_2}}_{>0} Y - L^s$$

Policy evaluation within the $IS - LM$ model

In a nutshell

- Short-run fluctuations away from the full-employment level are due to variations of aggregate demand.
- Public policies have room to counteract those short-run fluctuations.
- An expansionary fiscal policy may increase aggregate demand, but may be dampened by a crowding-out effect.
- An expansionary monetary policy increases aggregate demand if it does decrease the interest rate, which is not the case in a liquidity trap.

Macro-models: a compared approach

	Keynesian models (e.g. $IS - LM$)
Microeconomic foundations	missing
Agents' behaviour	not modelled
Macroeconomic relationships	postulated and estimated
Source of fluctuations	exogenous shocks

Macro-models: a compared approach

	Keynesian models (e.g. $IS - LM$)	DSGE models
Microeconomic foundations	missing	representative agent
Agents' behaviour	not modelled	maximisation, rational expectations
Macroeconomic relationships	postulated and estimated	mapping with individual behaviour
Source of fluctuations	exogenous shocks	exogenous shocks

Early DSGE models: background

Dissatisfaction with Keynesian economics:

- Empirical: stagflation in the 1970s: no trade-off between unemployment and inflation.
- Theoretical: how to get normative insights without utility functions and micro behaviors?
- Practical: the Lucas (1976) critique and the lack of policy-invariant parameters.

Main issues addressed:

- Positive aspect: what drives business cycles?
- Normative aspect: how should policy react to economic fluctuations?

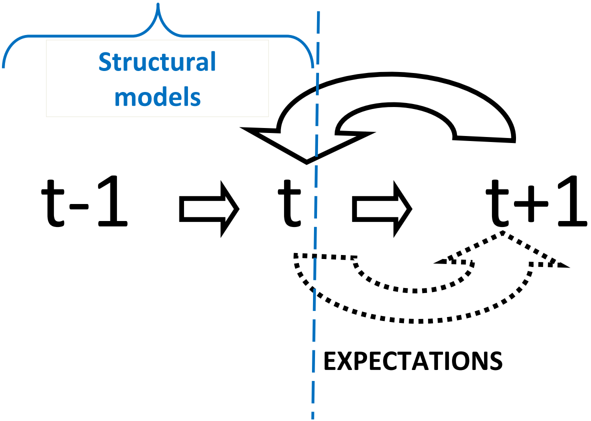
Main characteristics of DSGE models

- DSGE models have become a popular tool for policy analysis: Ex.: Bank of Canada (ToTEM), Bank of England (BEQM), European Central Bank (NAWM and EAGLE), Norges Bank (NEMO), Sveriges Riksbank (RAMSES), the US Federal Reserve (SIGMA), the IMF (GEM, GIMF and MAPMOD), the European Commission (QUEST III), Pessoa (Bank of Portugal).

- DSGE models are meant to capture plausible business cycle fluctuations.
- DSGE models are used for modeling policy scenarios, performing counterfactual experiments,....

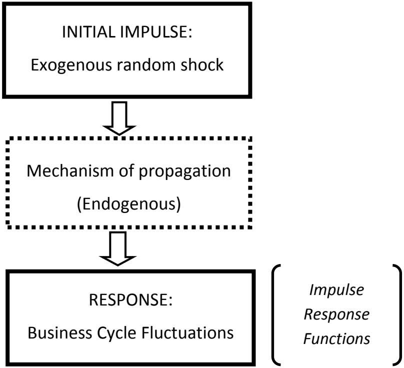
What are DSGE models?

Dynamic: agents' current choices depend on future uncertain expected outcomes



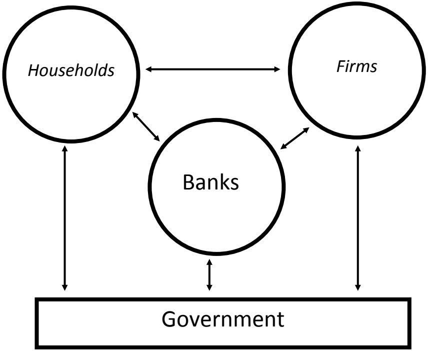
What are DSGE models?

Stochastic: random exogenous shocks affect the economy injecting uncertainty and generating economic fluctuations



What are DSGE models?

General Equilibrium: agents interact with each other



Main characteristics of DSGE models

DSGE models complement macro-structural models in several ways:

- Microfoundations
- Shocks have a clear economic interpretation
- **Role of expectations**: agents' current choices depend on future expected outcomes and perceived policy changes
- Models yield different reactions to (i) anticipated, (ii) unanticipated shocks or (iii) "news" and revision of expectations

Main ingredients of RBC models

- Real Business cycle models: **no role for money** (no money illusion) or any nominal variable.
- Real Business cycle models: neoclassical growth model + technology shocks. The internal mechanisms of the economy are stable, exogenous shocks move the economic variables around their stable path. to Solow model
- A **neoclassical** model: the First Welfare Theorem (complete markets, no friction, full information + price-taking behavior deliver a competitive equilibrium that is Pareto optimal), any market allocation is efficient, no role for short-run policy.
- A representative agent model with intertemporal decisions: firms and households.

Main ingredients of RBC models

Rational expectations:

- The representative agents are endowed with **substantive rationality** (by opposition to bounded rationality à la H. Simon). They are able to optimize their decisions (no cognitive limitations).
- They have **full information/perfect knowledge** about: i) the form of the model (objectives, constraints, symmetry between agents, law of motion of exogenous processes) and ii) the parameter values of the model.
- The expectation operator $E_t(\cdot) = E(\cdot / I_t)$ is the **best prediction possible** given the information set I_t available in period t. Agents make no **systematic** mistake; *on average*, their predictions are correct.
- Agents make **optimal decision plans conditional** on these predictions.

The households' consumption and labor schedule

Maximizes the expected value of the intertemporal utility function (with standard properties) over an infinite horizon:

$$\max_{C_{t+j},L_{t+j}} E_t \left(\sum_{j=0}^{\infty} \beta^j U(C_{t+j}, 1 - L_{t+j}) \right) \quad (1)$$

subject to

$$K_{t+1} = R_t K_t + L_t W_t - C_t$$

The households own the firms (capital is the only asset to invest/save in).

The Lagrangian program writes:

$$\max_{C_t,L_t,K_{t+1}} E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(U(C_t, 1 - L_t) + \lambda_t (R_t K_t + L_t W_t - C_t - K_{t+1}) \right) \right]$$

The households' consumption and labor schedule (con't)

Consider the following **functional form** for utility (separable CES):

$$U(C_t, 1-L_t) = \frac{\sigma}{\sigma-1} C_t^{\frac{\sigma-1}{\sigma}} + \frac{\phi}{\phi-1}(1-L_t)^{\frac{\phi-1}{\phi}},\ \sigma, \phi > 0.$$

We have:

$$\frac{\partial \mathcal{L}}{\partial C_t} : C_t^{\frac{-1}{\sigma}} = \lambda_t,$$

$$\frac{\partial \mathcal{L}}{\partial L_t} : (1-L_t)^{\frac{-1}{\phi}} = \lambda_t W_t,$$

and

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} : \lambda_t = \beta E_t[\lambda_{t+1} R_{t+1}]$$

with λ_t the marginal utility of consumption.

The households' consumption schedule (con't)

Equating the two (wrt C_t and k_{t+1}) gives the well-known **Euler equation** (**intertemporal substitution** of consumption):

$$\boxed{C_t^{\frac{-1}{\sigma}} = \beta E_t \left[R_{t+1} C_{t+1}^{\frac{-1}{\sigma}} \right]} \qquad (2)$$

The Euler equation states that:

- Agents smooth their consumption path over time (**permanent income hypothesis**, Friedman 1953).
- The role of **expectations is crucial in determining current consumption**.
- Current consumption does not depend on current income ($\simeq$ Keynesian economics).

The households' labor supply

Equating the two (wrt C_t and L_t) gives the relation of **intratemporal substitution between leisure and consumption**:

$$\boxed{W_t = \frac{(1 - L_t)^{-1/\phi}}{C_t^{-1/\sigma}}} \qquad (3)$$

Eq. (3) useful to study the income vs. substitution effects of a rise in wage on the consumption/leisure trade-off: a rise in W_t provokes a drop (income effect) or an increase in labor supply (substitution effect). In RBC models, **substitution** > **income effect**.

In the sequel, **for simplification**, an inelastic labor supply $L_t \equiv \bar{L}$.

The firms' labor and capital demands

$$\max_{K_t,L_t} \Pi_t = \max_{K_t,L_t} [Y_t - W_t L_t - (R_t - 1 + \delta) K_t] \tag{4}$$

with $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$. And the FOC give the equilibrium interest rate:

$$\frac{\partial \Pi_t}{\partial K_t} = 0 \Leftrightarrow \boxed{R_t = \alpha Z_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} + 1 - \delta} \tag{5}$$

and the equilibrium wage:

$$\frac{\partial \Pi_t}{\partial L_t} = 0 \Leftrightarrow \boxed{W_t = Z_t \left(\frac{K_t}{L_t}\right)^{\alpha} (1-\alpha) = (1-\alpha) Y_t} \tag{6}$$

The market clearing equations

(with inelastic labor supply $L_t = \bar{L} \equiv 1$)

1. Capital Demand (gross interest): $R_t = \alpha Z_t K_t^{\alpha-1} + 1 - \delta$
2. Labor Demand (wage rate): $W_t = (1-\alpha) Y_t$
3. Euler equation (consumption): $C_t^{\frac{-1}{\sigma}} = \beta E_t \left[R_{t+1} C_{t+1}^{\frac{-1}{\sigma}} \right]$
4. Production function (output): $Y_t = Z_t K_t^{\alpha}$
5. Capital accumulation (capital): $K_{t+1} = Y_t - C_t + (1-\delta) K_t$
6. And a specification for the process $\Omega_t = \{Z_j\}, j \leq t$.

We have 5 equations for 5 unknowns:

- **Exogenous variables** = stochastic processes $(\{Z_j\}_{j \geq 0})$
- **Endogenous variables**: **predetermined** = known at the beginning of period t ($K_t = S_{t-1}$); **non-predetermined variables** (R, W, C, Y)

Solving the model

1. The first step is to compute the **non-stochastic steady state**: obtained by assuming that all variables are constant (we drop the subscript t), and process $Z \equiv 1$:
   - Imposing $C_{t+1} = C_t = C^*$ in the Euler equation gives $R^* = \frac{1}{\beta}$;
   - Though the capital demand, we get K^*: $\alpha(K^*)^{\alpha-1} + 1 - \delta = 1/\beta$;
   - The wage W^* is then given by Y^* (marginal productivity of labor at K^*);
   - The capital accumulation equation for $K_{t+1} = K_t = K^*$ gives C^*.
2. The second step is to **log-linearize** the model around the steady state (local approximation).
3. The third step is to look for a **mapping $\mathcal{F}$ from the exogenous and predetermined variables into the endogenous variables**, given the initial conditions K_0 and Z_0: $[W_t, R_t, C_t] = \mathcal{F}(\Omega_t, K_t)$

Log-linearization around the steady state

Main idea: approximating the complicated, non-linear dynamics of the model in the neighborhood of the steady state by a linear function. More on log-linearization

The Euler equation

1. We replace X_t by $X^* e^{\tilde{x}_t}$: $C_t^{\frac{-1}{\sigma}} = \beta E_t \left[R_{t+1} C_{t+1}^{\frac{-1}{\sigma}} \right]$

$$\Leftrightarrow [C^* e^{\tilde{c}_t}]^{-1/\sigma} = \beta E_t [R^* e^{\tilde{R}_{t+1}}] E_t [C^* e^{\tilde{c}_{t+1}}]^{-1/\sigma}$$

2. Take the logs on both sides ($(C^*)^{-1/\sigma}$ drops):

$$\frac{-1}{\sigma} \tilde{c}_t = \ln \beta - \frac{1}{\sigma} E_t (\tilde{c_{t+1}}) + \ln R^* + E_t (\tilde{R}_{t+1})$$

3. Rearranging ($ln\beta + lnR^* = ln\beta + ln(1/\beta) = 0,\ R = 1 + r$):

$$\tilde{c}_t = E_t(\tilde{c_{t+1}}) - \sigma E_t(\tilde{R}_{t+1}) \Leftrightarrow \boxed{\tilde{c}_t = E_t(\tilde{c_{t+1}}) - \sigma \beta^{-1} E_t(\tilde{r}_{t+1})} \tag{7}$$

Log-linearization around the steady state

Production function

Replacing x by $x^* e^{\tilde{x}}$, dropping the steady state values, and taking the logs:

$$
\begin{aligned}
Y_t =& Z_t K_t^{\alpha} \text{ as } L_t \equiv 1 \\
&\Leftrightarrow \left[Y^* e^{\tilde{y}_t}\right] = Z^* e^{\tilde{z}_t} \left(K^* e^{\tilde{k}_t}\right)^{\alpha} \\
&\Leftrightarrow \boxed{\tilde{y}_t = \tilde{z} + \alpha \tilde{k}_t} \qquad (8)
\end{aligned}
$$

Technology process

In order to introduce persistence, the stochastic shock is usually a stable AR(1) process:

$$\boxed{\tilde{z}_{t+1} = \rho \tilde{z}_t + \epsilon_{t+1}} \qquad (9)$$

with $0 < \rho < 1$, and ϵ an idd shock with zero mean and variance σ_{ϵ}^2. The non-linear form of this process reads as follows:

$$Z_{t+1} = Z_t^{\rho} \mu_{t+1} \qquad (10)$$

with $Z^* = 1$, $\mu = 1 + \epsilon$ a stochastic process, $E(\mu) = 1$ and $\sigma_{\mu}^2 = \sigma_{\epsilon}^2$.

Log-linearization around the steady state

Capital accumulation

$$K_{t+1} = Z_t K_t^{\alpha} - C_t + (1-\delta)K_t = Y_t - C_t + (1-\delta)K_t$$
$$\Leftrightarrow K^* e^{\tilde{k}_{t+1}} = Y^* e^{\tilde{y}_t} - C^* e^{\tilde{c}_t} + (1-\delta)K^* e^{\tilde{k}_t}$$

With $K^* = Y^* - C^* + (1-\delta)K^*$ and $e^x \simeq (1+x)$ if x small enough:

$$\Leftrightarrow K^* \tilde{k}_{t+1} = Y^* \tilde{y}_t - C^* \tilde{c}_t + (1-\delta)K^* \tilde{k}_t$$
$$\Leftrightarrow \boxed{\tilde{k}_{t+1} = \frac{Y^*}{K^*}\tilde{y}_t - \frac{C^*}{K^*}\tilde{c}_t + (1-\delta)\tilde{k}_t} \qquad (11)$$

Labor demand

$$W_t = (1-\alpha)Y_t \Leftrightarrow W^* e^{\tilde{w}_t} = (1-\alpha)Y^* e^{\tilde{y}_t} \Leftrightarrow \boxed{\tilde{w}_t = \tilde{y}_t} \qquad (12)$$

Log-linearization around the steady state

Capital demand

$$R_t = Z_t \alpha K_t^{\alpha-1} + (1-\delta)$$
$$\Leftrightarrow \ln R^* + \frac{R_t - R^*}{R^*} = \ln\left(Z^* \alpha (K^*)^{\alpha-1} + (1-\delta)\right) +$$
$$\frac{\left[\alpha (K^*)^{\alpha-1}(Z_t - Z^*) + \alpha(\alpha-1) Z^* (K^*)^{\alpha-2}(K_t - K^*)\right]}{Z^* \alpha (K^*)^{\alpha-1} + (1-\delta)}$$

which easily simplifies into (in gross interest, with $R^* = 1/\beta$):

$$\boxed{\tilde{R}_t = \beta\alpha(K^*)^{\alpha-1}\ (\tilde{z}_t + (\alpha-1)\tilde{k}_t)} \tag{13}$$

Or, in interest **rate**:

$$\tilde{r}_t = \alpha(K*)^{\alpha-1}\ (\tilde{z}_t + (\alpha-1)\tilde{k}_t) \tag{14}$$

The linearized version of the model

(with inelastic labor supply, $L \equiv 1$)

1. Capital Demand (interest rate):
   $\tilde{r}_t = \alpha(K^*)^{\alpha-1}\ (\tilde{z}_t + (\alpha-1)\tilde{k}_t)$
2. Labor Demand (wage rate): $\tilde{w}_t = \tilde{y}_t$
3. Euler equation (consumption): $\tilde{c}_t = E_t(\tilde{c_{t+1}}) - \sigma\beta^{-1}E_t(\tilde{r}_{t+1})$
4. Production function (output): $\tilde{y}_t = \tilde{z}_t + \alpha\tilde{k}_t$
5. Capital accumulation (capital): $\tilde{k}_{t+1} = \frac{Y^*}{K^*}\tilde{y}_t - \frac{C^*}{K^*}\tilde{c}_t + (1-\delta)\tilde{k}_t$
6. And a specification for the process $\tilde{z}_{t+1} = \rho\tilde{z}_t + \epsilon_{t+1}$.

→ The associated steady state is zero for all variables.

→ All parameters in the equations depend on the structural parameters of the model (the preferences of the households σ and β, and the technology process α and ρ).

→ This system is only valid locally (in the neighborhood of the non-stochastic steady state).

Solving for the reduced form of the model

Main idea: Every DSGE model can be written in a reduced form:

$$S_{t+1} = \mathcal{M}(S_t, z_t, z_{t+1}) \tag{15}$$

- Recursive equilibrium: the decisions of all agents are optimal functions of a fixed set S of state variables and exogenous variables z.
- The map $\mathcal{M}(\cdot)$ represents the aggregate actual law of motion of the economy.
- The linearized version of $\mathcal{M}$ reads as:

$$\boxed{\tilde{s}_t = A\tilde{s}_{t-1} + B\epsilon_t} \tag{16}$$

  A and B matrices of parameters, ϵ a vector of disturbances, s the vector of state variables (in deviation from steady state).
- If all eigenvalues of A are within the unit circle, variables s are stationary, and the equilibrium of the model is determinate under rational expectations (i.e. unique and stable).

▸ more on stability

Solving for the reduced form of the model: the method of undetermined coefficients

1. Define the Minimum State Variable (MSV) solution:

$$\begin{cases} \tilde{c}_t = a_{ck}\tilde{k}_t + a_{cz}\tilde{z}_t \\ \tilde{k}_{t+1} = a_{kk}\tilde{k}_t + a_{kz}\tilde{z}_t \end{cases} \quad (17)$$

   with $\tilde{k}$ the predetermined endogenous variable, and $\tilde{z}$ the stochastic exogenous variable.
   The form of the equation for $\tilde{c}_t$ is based on the linearized Euler equation.
   The form of the equation for $\tilde{k}_{t+1}$ is obtained based on the equation for $\tilde{c}_t$ and the capital accumulation equation.

2. **Main idea**: Derive $\tilde{c}$ and $\tilde{k}$ as a function of the model's parameters from the model's equations and from the MSV solution. Then, equalize the two to get a_{ck}, a_{cz}, a_{kk} and a_{kz}.

Solving for the reduced form of the model: the method of undetermined coefficients

3. From the Euler equation (7) and the capital demand (14):

$$\Delta E_t(\tilde{c}_{t+1}) \equiv E_t(\tilde{c}_{t+1}) - \tilde{c}_t = \sigma\beta^{-1} E_t(\tilde{r}_{t+1}) \tag{18}$$

$$= \underbrace{\sigma\beta^{-1}\alpha(K^*)^{\alpha-1}}_{\lambda_c}\left[E_t(\tilde{z}_{t+1}) + (\alpha-1)\tilde{k}_{t+1}\right]$$

$$= \lambda_c \underbrace{E_t(\tilde{z}_{t+1})}_{\rho\tilde{z}_t} + \lambda_c(\alpha-1)\tilde{k}_{t+1} \tag{19}$$

with (from the capital accumulation equation (11)): (20)

$$\tilde{k}_{t+1} = \frac{Y^*}{K^*}\tilde{z}_t + \alpha\tilde{k}_t - \frac{C^*}{K^*}\underbrace{\tilde{c}_t}_{a_{ck}\tilde{k}_t + a_{cz}\tilde{z}_t} + (1-\delta)\tilde{k}_t$$

$$\Leftrightarrow \tilde{k}_{t+1} = \underbrace{\left(\frac{Y^*}{K^*} - \frac{C^*}{K^*}a_{cz}\right)}_{a_{kz}}\tilde{z}_t + \underbrace{\left(1-\delta+\alpha-\frac{C^*}{K^*}a_{ck}\right)}_{a_{kk}}\tilde{k}_t \tag{21}$$

Solving for the reduced form of the model: the method of undetermined coefficients

... which gives (putting (21) into (18)):

$$\begin{aligned}\Delta E_t(\tilde{c}_{t+1}) &= \lambda_c \rho \tilde{z}_t + \lambda_c(\alpha - 1)\left[\left(\frac{Y^*}{K^*} - \frac{C^*}{K^*}a_{cz}\right)\tilde{z}_t + \left(1 - \delta + \alpha - \frac{C^*}{K^*}a_{ck}\right)\tilde{k}_t\right] \\ &= \tilde{z}_t\left[\lambda_c \rho + \lambda_c(\alpha - 1)\left(\frac{Y^*}{K^*} - \frac{C^*}{K^*}a_{cz}\right)\right] \\ &+ \tilde{k}_t \lambda_c(\alpha - 1)\left(1 - \delta + \alpha - \frac{C^*}{K^*}a_{ck}\right) \qquad (22)\end{aligned}$$

4. From the MSV solution (17):

$$\Delta E_t(\tilde{c}_{t+1}) = a_{ck}\Delta \tilde{k}_{t+1} + a_{cz}\Delta E_t(\tilde{z}_{t+1}) = a_{ck}(\underbrace{\tilde{k}_{t+1}}_{=a_{kk}\tilde{k}_t + a_{kz}\tilde{z}_t \text{ (from the MSV solution)}} - \tilde{k}_t) + a_{cz}(\underbrace{E_t(\tilde{z}_{t+1})}_{\rho \tilde{z}_t} - \tilde{z}_t)$$

$$\Leftrightarrow \Delta E_t(\tilde{c}_{t+1}) = a_{ck}(a_{kk}\tilde{k}_t + a_{kz}\tilde{z}_t - \tilde{k}_t) + a_{ck}(\rho - 1)\tilde{z}_t$$

$$\Leftrightarrow \Delta E_t(\tilde{c}_{t+1}) = \tilde{z}_t\left(a_{ck}(\rho - 1) + a_{kz}\right) + \tilde{k}_t\left(a_{ck}(a_{kk} - 1)\right) \quad (23)$$

Solving for the reduced form of the model: the method of undetermined coefficients

5. Equalizing Steps 3 and 4 ((22) and (23)) gives:

$$\begin{cases} a_{ck}(\rho - 1) + a_{kz} = \lambda_c \rho + \lambda_c(\alpha - 1)\left(\frac{Y^*}{K^*} - \frac{C^*}{K^*} a_{cz}\right) \\ a_{ck}(a_{kk} - 1) = \lambda_c(\alpha - 1)(1 - \delta + \alpha - \frac{C^*}{K^*} a_{ck}) \end{cases}$$

6. and from the MSV solution (17) and Eq. (21):

$$\begin{cases} a_{kz} = \frac{Y^*}{K^*} - \frac{C^*}{K^*} a_{cz} \\ a_{kk} = 1 - \delta + \alpha - \frac{C^*}{K^*} a_{ck} \end{cases}$$

7. Quadratic equation in a_{ck} → 2 real solutions at most → we pick up the one within the unit circle for stability of the solution.
8. then we get a unique solution for a_{kk}, and then a_{cz} and a_{kz}.

Timing of events in the baseline RBC model

Intra-period timeline

1. The shock $\tilde{z}_t$ is realized.
2. With $\tilde{k}_t$ being know from period $t-1$, $\tilde{y}_t$, $\tilde{w}_t$, $\tilde{c}_t$ and $\tilde{r}_t$ are realized.
3. $\tilde{k}_{t+1}$ is realized.
4. etc.

Evaluating the model: visualizing the results

Main idea: comparing the simulated time series with actual ones.

- **Impulse response functions**: describe the response of the economy to a one-time exogenous shock in $\tilde{z}$;
- **Second moments of the data** (Monte Carlo simulations): compare standard deviations, autocorrelation up to several lags, cross-correlation patterns of the generated "time series.

To perform this exercise, RBC models are typically calibrated based on empirical estimation ($\alpha \simeq 1/3$).

Calibration (by opposition to estimation, as we will see in the second generation of DSGE models) allows us to separate the issue of parameter determination and model evaluation, but involves some discretionary choices.

Solving a DSGE model with Dynare (`http://www.dynare.org`)

What is Dynare? Why using Dynare?

- A free `matlab` extension to solve DSGE and OLG models (widely used in academia and public institution).
- Can incorporate many agent types (firms, consumers, financial intermediaries, government...).
- Can incorporate some heterogeneity (different classes of agents, such as Ricardian and "Rule-of-Thumb" consumers).
- Can incorporate different types of expectations (perfect foresight, RE, imperfect knowledge).
- Calibration or estimation of parameters.

Dynare Step 1: Installation

- Download Dynare version 4.3.3 from: http://www.dynare.org/download/dynare-stable
- Save it in C:/Programme Files/Dynare (or /Applications)
- In Matlab, `set path` → `add with subfolders...` and select the `Matlab` subdirectory of Dynare `C:/Programme Files/Dynare` (then `Save` and `Close`).
- A Dynare code has the extension `.mod`. See the subfolder `examples` in the Dynare folder.
- Make your own workspace in the Dynare folder (for instance, create a `advancedMacro` subfolder).
- To open your file: put matlab into the right directory (`cd c:\dynare\advancedMacro` or `cd /Applications/Dynare/4.4.3/advancedMacro`)

Dynare Step 2: Blocks in Dynare

- Tell Dynare what the variables, shocks and parameters of your model are.
- Ask Dynare to find the steady-state.
- Write down the model equations: **non-linear or log-linearized form**.
  (If in a non-linear form, Dynare (log)linearizes the model around the steady-state)
- Give the initial conditions.
- Run the simulations: impulse responses, variance decompositions, moments, etc...

Block 1: Declaration of variables and parameters

- Endogenous variables
- **var y, c, k, w, r, z;**
- Shocks (exogenous variables)
- **varexo e;**
- Parameters
- **parameters beta, alpha, delta, sigma, rho, kss, yss, css;**
  - $alpha = 0.3;$
  - ....;

Block 2: Model equations

- Solving for the steady state values
- Define a system in a separate .m file (matlab file) to solve for the steady state, and give initial values:
  solution = **fsolve(@ steadyStateSolve,x0,options,sigma, alpha, delta, beta)**

- Model declaration
  model;
  ....
  end;

If the model is linear, you need to specify: **model (linear);**

There needs to be as many equations as declared endogenous variables!

Notations in Dynare

- Time indexes are given in parenthesis:
  - X_{t+1} is written $X(+1)$
  - X_{t-1} is written $X(-1)$
  - X_t is written X (no time index needed)
  - $\mathbb{E}_t(X_{t+1}) = X(+1)$

$\rightarrow$ Capital is predetermined: k_{t+1} writes like k_t.

- Each equation has to end with semicolon (;)

- You can also comment out any line by starting the line with two forward slashes (%), or comment out an entire section by starting the section with $/*$ and ending with $*/$

Block 3: Introduction of shocks

1. Stochastic model
   - **shocks;**
   - **var e; stderr 0.1;**
   - **end;**

2. Deterministic model
   - **shocks;**
   - **var e;**
   - **periods 1:5;**
   - **values 0.1;**
   - **end;**

Block 4: Solving the model

- Initially, the model is at the steady-state. We want to evaluate what occurs after a **temporary shock**: how the economy goes back to the steady-state.
- Deterministic case: it is possible to compute numerical trajectories for the endogenous variables.
- Stochastic case: linearize the model, consider a linear approximation (Taylor expansion) of the non-linear model around the steady-state and compute the effects of small perturbations around steady-state.

Block 5: Simulations

1. To run a stochastic simulation:
   - `stoch_simul` **(order=1, irf=40) [list of variables];**
   - Give Dynare the list of variables that you want to analyze (by default all of them)
   - order=1 means that Dynare takes a first order Taylor approximation around the steady state (order=2 $\rightarrow$ second order approximation, etc...)
   - irf=40 means that you want Dynare to compute the impulse responses for 40 periods

2. To run a deterministic simulation:
   - **simul (periods=100);**

Dynare Step 3: Running the code

- Save the file as $*$.*mod*
  - For example: baselineRBC.mod
- To run the code: write in the Matlab command window **dynare baselineRBC**
- The Dynare toolbox reads your *.mod file and translates it in specific Matlab files
  - `baselineRBC.m`: main Matlab script for your model
  - `baselineRBC_static.m`: static model
  - `baselineRBC_dynamic.m`: dynamic model

Dynare Step 4: Dynare output (command window)

- The output is shown on the screen, and in separate figures (impulse responses)
- Output includes:
  - Policy and transition functions
  - Moments of the endogenous variables: mean, variance, standard deviation, skewness, kurtosis
  - Matrix of contemporaneous correlations
  - Coefficients of autocorrelation.
  - Impulse responses: check that it goes back to zero!
  - If you use the **check;** command: eigenvalues and a confirmation of the Blanchard-Kahn conditions

Dynare Step 4: Dynare output (workspace)

- Output is also stored in a separate structure `output_`.
- The structure output contains:
  - The steady state `output_.steady _state`
  - The variance-covariance matrix `output_.var`
  - The autocorrelations `output_.autocorr`
  - The impulse responses `output_.irfs`
  - The coefficients of the policy and transition functions `output_.dr`
  - Results (time paths of endogenous variables) from stochastic simulations `output_.endo_simul`

```
POLICY AND TRANSITION FUNCTIONS
                              y            c            k            w            r            z
Constant               3.449749     2.589794    34.398179     2.242337     1.010101     1.000000
k(-1)                  0.035101     0.062059     0.948042     0.022816    -0.000663            0
z(-1)                  3.104773     0.347191     2.757583     2.018103     0.031591     0.900000
e                      3.449748     0.385767     3.063981     2.242336     0.035101     1.000000
```

```
MODEL SUMMARY

  Number of variables:         6
  Number of stochastic shocks: 1
  Number of state variables:   2
  Number of jumpers:           2
  Number of static variables:  2
```

```
EIGENVALUES:
         Modulus             Real        Imaginary

             0.9              0.9                0
           0.948            0.948                0
           1.065            1.065                0
       1.092e+19       -1.092e+19                0


There are 2 eigenvalue(s) larger than 1 in modulus
for 2 forward-looking variable(s)

The rank condition is verified.
```

Example of the baseline RBC model (inelastic labor supply)

Getting macro statistics out of the model

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
y	3.4497	0.0970	0.0094
c	2.5898	0.0541	0.0029
k	34.3982	0.7851	0.6164
w	2.2423	0.0631	0.0040
r	1.0101	0.0007	0.0000
z	1.0000	0.0229	0.0005

MATRIX OF CORRELATIONS

Variables	y	c	k	w	r	z
y	1.0000	0.8181	0.7807	1.0000	0.5921	0.9714
c	0.8181	1.0000	0.9981	0.8181	0.0209	0.6582
k	0.7807	0.9981	1.0000	0.7807	-0.0414	0.6100
w	1.0000	0.8181	0.7807	1.0000	0.5921	0.9714
r	0.5921	0.0209	-0.0414	0.5921	1.0000	0.7664
z	0.9714	0.6582	0.6100	0.9714	0.7664	1.0000

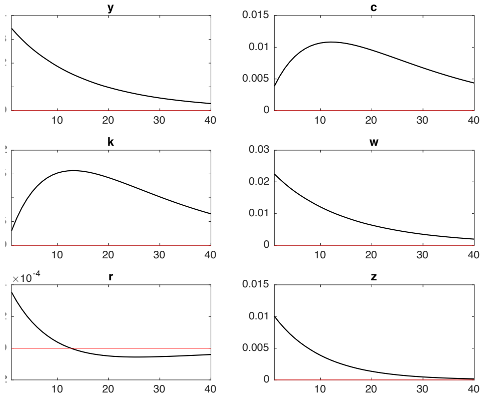
COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
y	0.9347	0.8741	0.8179	0.7656	0.7171
c	0.9959	0.9873	0.9748	0.9591	0.9407
k	0.9972	0.9896	0.9780	0.9630	0.9452
w	0.9347	0.8741	0.8179	0.7656	0.7171
r	0.8516	0.7205	0.6049	0.5031	0.4137
z	0.9000	0.8100	0.7290	0.6561	0.5905

- Relative volatility.
- Pro- and counter-cyclical patterns.
- Persistence.

Example of the baseline RBC model (inelastic labor supply)

Impulse responses to a positive technology shock



Co-movements of output, consumption, capital, wage and interest rates as a response to a positive technology shock.

Main limitations of RBC models

- Real shocks may be insufficient to explain aggregate economic fluctuations.
- Generate some counter-factual dynamics (e.g. consumption and hours work move in opposite direction because consumption and leisure are substitute, an increase in income generates a decrease in consumption!).
- Technology shocks are like "Deus ex machina": purely exogenous but explain everything (and what is a negative technology shock?)
- Interpretation of booms and recessions problematic: booms (resp. recessions) are times when households **choose** to work more (resp. less) at a response to temporary higher (resp. lower) wages due to a temporary better (resp. worse) technology.
- Policy recommendation: no room for policy intervention. **Fluctuations** are an **efficient** response of agents.

The Solow growth model (1956): ingredients

An **aggregate, neoclassical** model of capital accumulation that predicts a balanced, **stable growth path in the long run**.

- Full employment L_t, Cobb Douglas production function:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \Rightarrow y_t = Z_t k_t^{\alpha} (\text{per capita}) \tag{24}$$

- Fixed saving rate s: $C_t = (1-s)Y_t$, $S_t = sY_t$, $Y_t = C_t + S_t$ and $I_t = S_t$ (no financial market, no trade, no government):

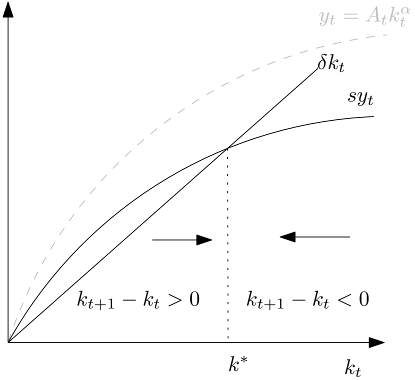
$$K_{t+1} = (1-\delta)K_t + I_t \tag{25}$$

- In per capital terms, we have:

$$\Delta k_t \equiv k_{t+1} - k_t = sZ_t k_t^{\alpha} - \delta k_t \tag{26}$$

with Z_t the GFP, K_t the stock of capital, Y_t the output, $0 < \alpha < 1$, C_t consumption, S_t savings, I_t investment, $0 < \delta < 1$ the depreciation rate of capital.

The Solow growth model (1956): dynamics

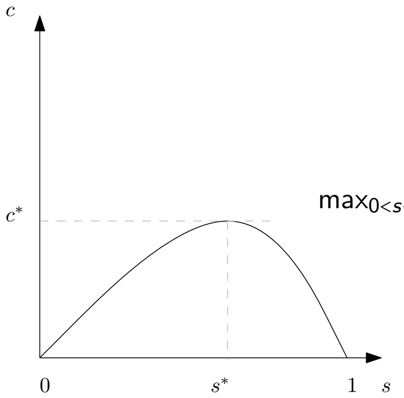


$$\Delta k_t \equiv k_{t+1} - k_t = s \underbrace{k_t^\alpha}_{y_t} - \delta k_t$$

Assuming $Z_t \equiv 1$:
Steady state such as $k_{t+1} = k_t = k^* \Leftrightarrow k^* = \left(\frac{s}{\delta}\right)^{\frac{1}{1-\alpha}}$
(and then compute y^*, c^*, etc. as a function of k^*).

The economy converges to a balanced growth path.

The Solow growth model (1956): optimal saving rate



$$\max_{0<s<1} c^*(s) : \frac{\partial c^*}{\partial s} = 0 \Leftrightarrow \boxed{s^* = \alpha}$$

back Trade-off between savings and consumption.

Notes on log-linearization

- We want to express all variables in percentage deviation from their steady state values:

$$\boxed{\tilde{x}_t \equiv \frac{X_t - X^*}{X^*}} \Leftrightarrow X_t = X^*(1 + \tilde{x}_t)$$

- If $\tilde{x}_t$ is **small enough**, recall $ln(X+1) \approx X$, and we have:

$$\ln X_t = \ln X^* + \ln(1 + \tilde{x}_t) \simeq \ln X^* + \tilde{x}_t \Leftrightarrow \boxed{\tilde{x}_t = ln(X_t) - \ln X^*}$$

- Recall the Taylor expansion of a time series around some value x^*:

$$f(x) \approx f(x^*) + \frac{f'(x^*)}{1!}(x - x^*) + \frac{f^2(x^*)}{2!}(x - x^*)^2$$
$$+ \frac{f^3(x^*)}{3!}(x - x^*)^3 + \ldots + o(\| x - x^* \|^n)$$

Notes on log-linearization (con't)

- In DSGE models, we usually use a first-order Taylor expansion:

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*)$$

Or, in two (or more) dimensions:

$$f(x,y) \approx f(x^*,y^*) + f'_x(x^*,y^*)(x - x^*) + f'_y(x^*,y^*)(y - y^*)$$

- Log-linearizing an equation **is taking the log and then a first-order Taylor expansion**:

$$\ln f(x) \approx \ln f(x^*) + \underbrace{\frac{f'(x^*)}{f(x^*)}}_{\frac{\partial \ln f(x)}{\partial x}|x=x^*} (x - x^*) \Leftrightarrow \frac{f'(x^*)}{f(x^*)}(x - x^*) \approx f(\tilde{x}_t)$$

- Or replacing all variables by $X_t = x^* e^{\tilde{x}_t}$, taking the logs and simplifying (with $e^{\tilde{x}} \simeq 1 + \tilde{x}$). back

Notes on stability: Blanchard & Kahn (1980) conditions

Any DSGE model can be written as:

$$\begin{bmatrix} K_{t+1} \\ E_t(X_{t+1}) \end{bmatrix} = A \begin{bmatrix} K_t \\ X_t \end{bmatrix} + \Gamma Z_t$$

with K a vector of predetermined variables, X a vector of endogenous, non-predetermined variables, Z a vector of exogenous disturbances, A and Γ are vectors of parameters.

The **local** stability of the system (i.e. whether, dynamically, it converges towards a steady state for a set of initial conditions) depends on the matrix A.

Notes on stability: Blanchard & Kahn (1980) conditions

- The equilibrium of the system is **stable** if the number of unstable eigenvalues (i.e. outside the unit circle) of A is equal to the number of non-predetermined variables.
- If the number of unstable eigenvalues of A exceeds the number of non-predetermined variables, there is **no non-explosive solution.**
- If the number of unstable eigenvalues of A is less than the number of non-predetermined variables, there is an **infinity of solutions** (multiplicity, indeterminacy). back